**Mathematics Task Arcs**

**Overview of Mathematics Task Arcs:**

A task arc is a set of related lessons which consists of eight tasks and their associated lesson guides. The lessons are focused on a small number of standards within a domain of the Common Core State Standards for Mathematics. In some cases, a small number of related standards from more than one domain may be addressed.

A unique aspect of the task arc is the identification of essential understandings of mathematics. An essential understanding is the underlying mathematical truth in the lesson. The essential understandings are critical later in the lesson guides, because of the solution paths and the discussion questions outlined in the share, discuss, and analyze phase of the lesson are driven by the essential understandings.

The Lesson Progression Chart found in each task arc outlines the growing focus of content to be studied and the strategies and representations students may use. The lessons are sequenced in deliberate and intentional ways and are designed to be implemented in their entirety. It is possible for students to develop a deep understanding of concepts because a small number of standards are targeted. Lesson concepts remain the same as the lessons progress; however the context or representations change.

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Understanding and Solving Systems of Linear Equations

A SET OF RELATED LESSONS

Grade 8
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Introduction
Understanding and Solving Systems of Linear Equations
A SET OF RELATED LESSONS
Overview

In this set of related lessons, students extend their previous understanding of linear equations to include systems of linear equations. Students develop and solidify an understanding of what it means to be a solution to a system, how to solve a system using multiple representations, and the conditions under which a system has one, zero, or infinitely many solutions.

In Task 1, students explore what it means to be a solution to a system of equations using various representations. In Task 2, they build on this understanding by solving a linear system of equations in a context using various strategies and/or representations. In Task 3, students consider a system of linear equations in a table of values and then analyze how to solve a system of equations algebraically by applying the properties of equality. In Task 4, students solidify their understanding of what it means to be a solution and how to solve systems of linear equations.

Task 5 is an open-ended exploration of the conditions under which a system has one, zero, or infinitely many solutions. In Tasks 6 and 7, students continue to develop this understanding in various contexts. In Task 8, students solidify their understanding of the conditions under which systems of linear equations have one, zero, or infinitely many solutions. The tasks are aligned to the 8.EE.C.8a and 8.EE.C.8b Content Standards of the Common Core State Standards for Mathematics.

The prerequisite knowledge necessary to enter these lessons is an understanding of linear equations and their solutions using multiple representations.

Through engaging in the lessons in this set of related tasks, students will:

- analyze what it means to be a solution to a system of equations;
- solve a system of equations graphically, using a table of values and algebraically; and
- develop an understanding of the conditions under which a system of equations has one solution, zero solutions, and infinitely many solutions.

By the end of these lessons, students will be able to answer the following overarching questions:

- What does it mean to be a solution to a system of equations?
- How is a solution to a system of equations represented in multiple representations?
- What strategies are used to solve a system of equations?

The questions provided in the guide will make it possible for students to work in ways consistent with the Standards for Mathematical Practice. It is not the Institute for Learning’s expectation that students will name the Standards for Mathematical Practice. Instead, the teacher can mark agreement and disagreement of mathematical reasoning or identify characteristics of a good explanation (MP3). The teacher can note and mark times when students independently provide an equation and then re-contextualize the equation in the context of the situational problem (MP2). The teacher might also ask students to reflect on the benefit of using repeated reasoning, as this may help them understand the value of this mathematical practice in helping them see patterns and relationships (MP8). In study groups, topics such as these should be discussed regularly because the lesson guides have been designed with these ideas in mind. You and your colleagues may consider labeling the questions in the guide with the Standards for Mathematical Practice.
## Identified CCSSM and Essential Understandings

### CCSS for Mathematical Content: Expressions and Equations

<table>
<thead>
<tr>
<th>Essential Understandings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Analyze and solve linear equations and pairs of simultaneous linear equations.</td>
</tr>
</tbody>
</table>

### Essential Understandings

1. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

2. The solution to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs \((x, y)\) that make the equation a true statement or satisfy the equation.

3. The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

4. The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

5. The solution to a system of two or more linear equations can be represented algebraically, graphically, in a table, and in context.
### CCSS for Mathematical Content: Expressions and Equations

<table>
<thead>
<tr>
<th>8.EE.C.8b</th>
<th>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Essential Understandings</td>
<td>Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution. Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions. Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions. Because both the x-value and the y-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.</td>
</tr>
</tbody>
</table>

### The CCSS for Mathematical Practice²

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

---

### Tasks’ CCSSM Alignment

<table>
<thead>
<tr>
<th>Task</th>
<th>8.EE.C.8a</th>
<th>8.EE.C.8b</th>
<th>MP 1</th>
<th>MP 2</th>
<th>MP 3</th>
<th>MP 4</th>
<th>MP 5</th>
<th>MP 6</th>
<th>MP 7</th>
<th>MP 8</th>
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</thead>
<tbody>
<tr>
<td><strong>Task 1</strong> Water Tanks</td>
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<tr>
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<tr>
<td><strong>Task 3</strong> Scuba Math</td>
<td>✓</td>
<td>✓</td>
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<td><strong>Task 4</strong> Income &amp; Expenses</td>
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<td><strong>Task 5</strong> To Meet or Not to Meet?</td>
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<tr>
<td><strong>Task 6</strong> Savings Plans</td>
<td>✓</td>
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<tr>
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<tr>
<td><strong>Task 7</strong> Pick Your Programs</td>
<td>✓</td>
<td>✓</td>
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<tr>
<td><strong>Task 8</strong> Still Stretching: Yoga Plans Revisited</td>
<td>✓</td>
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</tbody>
</table>
# Lesson Progression Chart

## Overarching Questions
- What does it mean to be a solution to a system of equations?
- How is a solution to a system of equations represented in multiple representations?
- What strategies are used to solve a system of equations?

<table>
<thead>
<tr>
<th>Task 1: Water Tanks</th>
<th>Task 2: Yoga Plans</th>
<th>Task 3: Scuba Math</th>
<th>Task 4: Income &amp; Expenses</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Content</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Develops understanding of what it means to be a solution to a system of equations.</td>
<td>Continues developing an understanding of what it means to be a solution to a system of equations; begins developing strategies for solving a system of equations.</td>
<td>Continues developing various strategies to solve a system of equations; focuses on the development of an algebraic strategy.</td>
<td>Solidifies what it means to be a solution to a system of equations, as well as strategies for solving a system of linear equations.</td>
</tr>
<tr>
<td><strong>Strategy</strong></td>
<td></td>
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</tr>
<tr>
<td>Extends the lines to determine the point of intersection; creates a table of values; uses guess-and-check.</td>
<td>Graphs the lines to determine the point of intersection; substitutes values into the equations; creates a table of values.</td>
<td>Extends the table; uses equations or a graph; in part two, uses the properties of equality to isolate x as an algebraic strategy.</td>
<td>Solves the equations algebraically; extends the table; graphs and determines the point of intersection.</td>
</tr>
<tr>
<td><strong>Representations</strong></td>
<td></td>
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<tr>
<td>Starts with equations and part of a graph; students use the graph, equations, and/or table of values.</td>
<td>Starts with a context; students use the equations, graph, and/or a table of values.</td>
<td>Starts with a context and table of values; students use the tables, graph, or algebraic strategy. In part two, students focus on an algebraic strategy.</td>
<td>Starts with a context and equations; students use the equation.</td>
</tr>
<tr>
<td>Task</td>
<td>Content</td>
<td>Strategy</td>
<td>Representations</td>
</tr>
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<tr>
<td>Task 5  &lt;br&gt;To Meet or Not to Meet?  &lt;br&gt;Developing Understanding</td>
<td>Investigates the conditions under which a system of linear equations has one solution, zero solutions, or infinitely many solutions.</td>
<td>Determines the point of intersection graphically; manipulates the equations algebraically using the properties of equality.</td>
<td>Starts with a graph and moves to equations.</td>
</tr>
<tr>
<td>Task 6  &lt;br&gt;Savings Plans  &lt;br&gt;Developing Understanding</td>
<td>Continues developing an understanding of the conditions under which a system of equations has one solution.</td>
<td>Creates a table, graph, or uses the structure of the equations.</td>
<td>Starts with a context and a table and moves to equations, graphs, and a table of values.</td>
</tr>
<tr>
<td>Task 7  &lt;br&gt;Pick Your Programs  &lt;br&gt;Developing Understanding</td>
<td>Continues developing an understanding of what it means for a system of equations to have one, zero, or infinitely many solutions.</td>
<td>Makes sense of the problem graphically, analyzes the structure of the equations; uses numeric methods.</td>
<td>Starts with a context and moves to equations, graphs, and a table of values.</td>
</tr>
<tr>
<td>Task 8  &lt;br&gt;Still Stretching: Yoga Plans Revisited  &lt;br&gt;Solidifying Understanding</td>
<td>Solidifies an understanding of the conditions under which a system of equations has one solution, zero solutions, or infinitely many solutions.</td>
<td>Makes sense of the problem graphically and algebraically.</td>
<td>Starts with a context and a graph, moves to algebraic representations.</td>
</tr>
</tbody>
</table>
Tasks and Lesson Guides
Understanding and Solving Systems of Linear Equations
A SET OF RELATED LESSONS
The graph below shows the amount of water in two different tanks, T and W, over a period of time. T can be represented by the equation: \( y = 900 - 50x \) while W can be represented by the equation: \( y = 300 + 25x \). (Assume that the rates of water loss and water gain continue as shown.)

1. Label each line with the appropriate equation. Then describe what each constant and variable represents in this problem situation.
2. At what point do the lines intersect? What does this point represent in the context of this problem situation?

3. Use the equations or a table of values to verify the point of intersection. Explain your reasoning.
**Water Tanks**

**Rationale for Lesson:** Given a partial graph and a system of equations, students are asked to reason about the meaning of the equations in the context of a real-life situation. Students will make sense of the solution to this system of equations as an introduction to solving systems of linear equations.

**Task 1: Water Tanks**

The graph below shows the amount of water in two different tanks, T and W, over a period of time. T can be represented by the equation: \( y = 900 - 50x \) while W can be represented by the equation: \( y = 300 + 25x \). (Assume that the rates of water loss and water gain continue as shown.)

1. Label each line with the appropriate equation. Then describe what each constant and variable represents in this problem situation.
2. At what point do the lines intersect? What does this point represent in the context of this problem situation?
3. Use the equations or a table of values to verify the point of intersection. Explain your reasoning.

See student paper for graph.

| Common Core Content Standards | 8.EE.C.8a | Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
| 8.EE.C.8b | Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, \( 3x + 2y = 5 \) and \( 3x + 2y = 6 \) have no solution because \( 3x + 2y \) cannot simultaneously be 5 and 6.

| Standards for Mathematical Practice | MP1 Make sense of problems and persevere in solving them.
| MP2 Reason abstractly and quantitatively.
| MP4 Model with mathematics.
| MP5 Use appropriate tools strategically.
| MP6 Attend to precision.
| MP7 Look for and make use of structure.

| Essential Understandings | The solutions to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs \((x, y)\) that make the equation a true statement or satisfy the equation.
| The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.
| The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

| Materials Needed | Student reproducible task sheet
| Straight edge, graph paper, calculators (optional)
SET-UP PHASE

I’d like a student to read the task aloud while everybody else reads along silently. Without giving away the solution, can someone explain the problem context? I’m going to give you 5-10 minutes to work before sharing ideas with the group. Graph paper, calculators, and rulers are available at your tables to use as needed.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Group can’t get started.</strong></td>
<td>What do you know about each water tank?</td>
<td>What will the graphs look like if we extend them, since it says we can assume that the rates of water loss and water gain in both tanks are constant?</td>
</tr>
<tr>
<td><strong>Continues graph with a straight edge.</strong></td>
<td>How did you know that you could extend each line this way? How can you use the graph to make sense of this problem?</td>
<td>How can you know for sure that this point of intersection is an accurate solution to the problem? Is there another representation of this data that we can use to verify the point of intersection?</td>
</tr>
<tr>
<td><strong>Makes a table of values.</strong></td>
<td>How did you know what values to use in your table? How can you use the table to make sense of this problem?</td>
<td>How can you use your graph or the equations to decide if your solution is an accurate one?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>T</th>
<th>yTank T</th>
<th>yTank W</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>900</td>
<td>300</td>
</tr>
<tr>
<td>1</td>
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<tr>
<td>8</td>
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</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

EU: The solutions to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs \((x, y)\) that make the equation a true statement or satisfy the equation.

- Who can explain how you know which line represents Tank T and which line represents Tank W? (Tank T has a negative slope and Tank W has a positive slope.)
- Who can say more? (Tank T starts at 900 and Tank W starts at 300.)
- So we can make sense of the graphs by the rate of change and the amount of water at \(t = 0\).
  (Marking)
- Let’s examine the lines on the graph. What does each point on the lines represent? (A solution.)
  - Who can add on to that? (Each point on the line represents a solution to that equation.)
  - Who can explain what he meant by “that equation”? What equation? (We know the equation for each line. If a point is on a line, it has to be a solution to the equation of that line.)
  - Remind me. What does it mean to be a solution to an equation? (The answer.)
  - Who can say more? (A solution makes it true.) (A solution makes its equation true.)
  - Who understood what she just said and can repeat it in your own words? (Well, if it’s a solution to an equation, you can plug it in and get a true statement.)
  - But we are talking about points on a line here. How do we “plug in” points on a line? What does that mean? (Challenging) (It means we plug into the equation the x-value for x and the y-value for y, and, if we get a true statement, it’s a solution to the equation.)
  - So you are saying that we can substitute the x- and y-values of the point into the associated equation and the resulting equation is a true statement. (Revoicing)
  - Stop and jot. One of our equations is \(y = 900 - 50x\). Show me how you know that these two points \((2, 800)\) and \((4, 700)\) are solutions to the equation (pointing to the points on the graph).
  - So then, how many solutions are there for each line? (Lots.)
  - Why do you say “lots”? (All the points on that line work.) (There are lots of points on that line.) (Infinitely many.)
  - Okay, so I’m hearing that a line on a graph represents an equation, and the line is made up of infinitely many points. These points are all solutions to the equation. If you substitute any point on the graph into the equation, then a true statement results. (Recapping)

EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

EU: The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

Uses the graph:

- Who can show me how they made sense of the problem and determined a solution using the graphs? Can I have someone show us what their graph looks like when they extended the lines? How did we know that the lines were straight and could continue the same way? (It said we could assume that the water continued to go in or come out at the same rate, so it’s linear.)

continued on next page
(Student name), please repeat that in your own words. (Okay—same rate is constant rate. Linear equations have a constant rate. So draw a line.)

(Student name), your group drew a line. Was that your thinking, too? Come show us your work. (Sort of. But we drew slope triangles first, so we would get a better line.)

A better line? Who understands what she means and can repeat it for us? (Well, if you just draw lines, you may not draw them accurately. The slope triangles give you lots of other points on the line, so the line is better or more accurate.)

Who can remind us about where those slope triangles came from? How did this group know to draw them as they did?

So some of you extended the lines and others drew slope triangles to extend the lines. Are the lines the solution to the problem? (No. The question asks about the point of intersection.)

Who thinks they know what it means to be a point of intersection? (It is where they cross. Like a street intersection. Where they cross.) Who can explain what this point of intersection represents? (Where the lines come together or cross.)

So I hear you saying it is where the lines cross; what can you say about the (x, y) values at this point? (They are the same.)

Who can add on to that? (The point is a solution to each equation.)

This is very important—so the point of intersection represents a solution to each equation.

(Marking)

Who can show us how they determined this point of intersection?

What is the point of intersection? What do these numbers tell us about this situation? (8, 500) At 8 hours, both tanks will have 500 gallons of water.

If you used your graph to find the point of intersection, were you positive that this was the exact point? (No. We checked. Ours was wrong.)

How do you check to see if you are correct? (You substitute your x in for the number of hours and make sure that it equals 500 gallons at this point.)

Can someone add on? (It’s like what we said before. If (8, 500) is the real intersection point, it makes both equations true when you substitute.)

This is an important point. Who can restate this in their own words using the graph and the equation? (If you plug in the x-value, you get the same y-value from the equations. For both lines, when x = 8 you look at the graph and find the y-value and see that y = 500.)

Is there more than one point? (No, there is only one point where they cross.) Say more; how do you know they won’t cross again? (How can two lines cross again? Can’t happen.) (Because the lines continue in opposite directions at the same rate.)

Uses a table:

Group C represented this problem using a table of values. Please explain your thinking.

How do we know that we have found the point of intersection when we look at the table of values for this situation? (The time and amount of water would be the same for both tanks.)

Okay, so when we look at the graph, we look for the point where the lines cross, though depending on how precise you were with your straight edge, this point may vary. What are we looking for in a table when we want to find the point of intersection? (Look for the water amount values to get closer together and find when they are the same.)

Will this be the only point of intersection? How do you know by looking at the table? (The values after you pass the point of intersection start to get farther apart.)

And how can I verify that this is the correct point of intersection?
• So I hear you saying that the point of intersection is the point that, when substituted into the equations, will make both equations true. I am also hearing you say that we can find that point of intersection using a table or graph, and check the point out using the equations. (Recapping)

• The point (4, 700) is on one of the lines. How can we use the equations to check to make sure that this is not the point of intersection? (When you substitute the values into each equation, the equation for Tank T is true, but Tank W is not.)

• The solution to the system looks different with different representations. Can someone restate how we know we have found the solution with a table, graph, or the equations?

• Okay, so I’m hearing that there is only one solution to the system of equations and that it is the point where the number of hours the tank has been pumping AND the amount of water in both tanks is the same. You can also see this value in the table where the values are the same and in the equation where the output is the same for a given input. (Recapping and Marking)

### Application
What is the point of intersection for the graphs of the equations $y = 2x + 5$ and $y = -5x - 12$? How can you use the equations to check to make sure your point is accurate?

### Summary
What does the solution to a system look like on a graph or table? How can you use your equations to verify your solution?

### Quick Write
No Quick Write for students.

**Support for students who are English learners (EL):**
1. Bring in or display images of a water tank so that students identified as English learners associate the words in the problem with the images.
2. On a wall section in the room, create an English term/appropriate native language term (if it exists)/definition/picture for math terms relevant to the unit. For this task, make sure “point of intersection” and “solution to a system” are posted.
3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can rehear and reprocess the day’s key ideas.
4. Begin to create a poster titled, Strategies for Solving Systems of Equations. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
Yoga Plans

Flexible Yoga charges a base rate of $35 to join their yoga studio, plus $4.50 per yoga class. A competing yoga studio, Stretch-It-Out, charges a base rate of $27.50 to join and $5.25 per class.

How many classes would you have to take before joining Flexible Yoga costs the same amount of money as Stretch-It-Out? Explain your reasoning. Use equations to support your answer.
## Yoga Plans

**Rationale for Lesson:** Given a context, students are asked to determine a solution to a system of linear equations. Students may use a table, graph, or equations to determine the solution, but must verify their answer using equations.

### Task 2: Yoga Plans
Flexible Yoga charges a base rate of $35 to join their yoga studio, plus $4.50 per yoga class. A competing yoga studio, Stretch-It-Out, charges a base rate of $27.50 to join and $5.25 per class.

How many classes would you have to take before joining Flexible Yoga costs the same amount of money as Stretch-It-Out? Explain your reasoning. Use equations to support your answer.

### Common Core Content Standards

<table>
<thead>
<tr>
<th>Standard</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.EE.C.8a</td>
<td>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
</tr>
<tr>
<td>8.EE.C.8b</td>
<td>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</td>
</tr>
</tbody>
</table>

### Standards for Mathematical Practice

<table>
<thead>
<tr>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>MP1</td>
<td>Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>MP2</td>
<td>Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>MP4</td>
<td>Model with mathematics.</td>
</tr>
<tr>
<td>MP5</td>
<td>Use appropriate tools strategically.</td>
</tr>
<tr>
<td>MP6</td>
<td>Attend to precision.</td>
</tr>
<tr>
<td>MP7</td>
<td>Look for and make use of structure.</td>
</tr>
</tbody>
</table>

### Essential Understandings

- The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs $(x, y)$ that make both equations true statements or satisfy the equations simultaneously.
- The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

### Materials Needed

- Student reproducible task sheet
- Straight edge, graph paper, calculators (optional)
SET-UP PHASE
I’d like a student to read the task aloud while everybody else reads along silently. Can someone explain the problem in his/her own words? Has anyone ever taken a yoga class? How is a yoga studio the same/different than a gym? How do these yoga companies charge their customers? If you were planning to join Flexible Yoga or Stretch-It-Out why would knowing the number of classes you might take be important to you? Let’s begin working independently before discussing with a partner. Graph paper, calculators, and rulers are available at your tables to use as needed.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group can’t get started.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Makes a table of values.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>x</th>
<th>y Flexible</th>
<th>y Stretch-it-Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>35</td>
<td>27.50</td>
</tr>
<tr>
<td>1</td>
<td>39.50</td>
<td>32.75</td>
</tr>
<tr>
<td>2</td>
<td>44</td>
<td>38</td>
</tr>
<tr>
<td>3</td>
<td>48.50</td>
<td>43.25</td>
</tr>
<tr>
<td>4</td>
<td>53</td>
<td>48.50</td>
</tr>
<tr>
<td>5</td>
<td>57.50</td>
<td>53.75</td>
</tr>
<tr>
<td>6</td>
<td>62</td>
<td>59</td>
</tr>
<tr>
<td>7</td>
<td>66.50</td>
<td>64.25</td>
</tr>
<tr>
<td>8</td>
<td>71</td>
<td>69.50</td>
</tr>
<tr>
<td>9</td>
<td>75.50</td>
<td>74.75</td>
</tr>
<tr>
<td>10</td>
<td>80</td>
<td>80</td>
</tr>
</tbody>
</table>

Explain how you created your table of values. How will this help you make sense of the problem?

What is the problem asking you to find? How can you use your table to answer the question?
### Possible Student Pathways

<table>
<thead>
<tr>
<th>Makes a graph.</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Graph Image" /></td>
<td>Why did you construct your graph? What does it tell you?</td>
<td>How can you use your graph to help you make sense of the problem?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Guesses and checks after writing equations.</th>
<th>How did you determine your equations? What are you trying to do with these calculations?</th>
<th>What are you looking for? Is there a representation that can help you organize your work more efficiently so you can solve the problem?</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y = 35 + 4.5x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 27.50 + 5.25x$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 35 + (4.5)(5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 57.50$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 27.50 + (5.25)(5)$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y = 53.75$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$57.50 ≠ 53.75$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

**EU:** The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

**EU:** The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

**Uses a table:**
- Who can tell us about how they made sense of this problem? What ways can we mathematically represent this situation? *(Table of values, graph, and equations.)*
- Group A, you represented this problem using a table. Tell us about how you created the table and how your table represents the problem situation.
- Who can add on to this? How does the table help you make sense of what the problem is asking? *(We’re trying to figure out when they are the same. You can see the one starts higher but the other keeps getting closer and closer until they are the same right here.)*
- Who can restate this in his or her own words?
- What are we looking for in the table? *(When they cost the same amount of money.)*
- Who agrees? Disagrees? Okay, I have a question for you. Don’t they cost the same amount of money here and here (pointing to the $48.50 at 3 and 4 hours)?
- Turn and Talk. Is $48.50 another solution to the problem? Why or why not? *(Challenging)*
- (Student name), what do you and your partner think? *(Well, they are the same amount of money. But I don’t think that’s enough. The $80.00 is the better answer.)*
- Why? What are we looking for here? *(The point of intersection.)*
- Okay, but then we need a graph. Let’s check in with a group who used a graph and see what light the graph sheds on our table and the question about the $48.50.

**Uses a graph:**
- Group B, you used a graph. Tell us about how you created the graph and how this represents the problem situation. *(We plotted the points. We determined the cost for one class and plotted the point here. Two classes and plotted this here, etc. We did this for both yoga studios.)*
- How do the points you plotted compare with the data in Group A’s table? *(We plotted mostly those points.)*
- So does that mean that both the table and the graph represent the Yoga Plans’ context?
- Do both representations have the same solution? *(Yes. Both are the same at $80.)*
- Why $80? *(Because it’s the point of intersection.)*
- What does this point represent? *(It represents when they cost the same amount of money.)*
- But the cost is also the same at $48.50. So let’s look at this a bit differently. Look at both the $48.50 and the $80. Jot down some similarities about those two values as they appear in the table and graph, and some differences.
- (Student name), please share some of your thoughts. *(Same: cost. Different: $48.50 is not the point of intersection.)*
- Who can add on? Who can say what is different about that point of intersection from the situations with $48.50? *(It’s also the point where the number of classes is the same.)* I hear you saying that both the number of classes AND the cost is the same at the point of intersection. *(Marking)* Is that also true of the $48.50?
- Say more. Why is the number of classes and the cost the same only at the point of intersection? *(Because this point is on both of the lines so it is a solution to both of the equations.)*
EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

- I heard the class say earlier that we can use several representations to make sense of this problem. If you used your graph to find the point of intersection, how can you be positive that \((10, 80)\) is the exact point? (We used equations to check the point of intersection.)

Uses an equation:
- Then let’s take a look at Group C’s work. Group C, explain how you created the equations and how they represent the problem situation.
- I’m hearing you say that we can use equations to verify the point of intersection. (Marking) Before we go into how to verify the point of intersection, can somebody first tell us about the equations?
- Who can tell us what the constant values and coefficients in their equations represent in this problem? (The constant values represent the base fee and then this coefficient is the cost per class.)
- Where do these values appear on the graph?
- Now let’s go back to how you can use your equations to make sure the point of intersection is correct. (You substitute 10 in for the number of classes and make sure that both equations equal the same amount at this point. In this case, it’s $80.)
- This is an important point. Who can restate this in their own words? (They need to be the same cost for the same number of classes.)
- How can we use these equations to see if both $48.50 and $80.00 are solutions? (We can plug in the \(x\) and see if it makes the equation true.)
- Which equation? (Both equations.)
- Okay, go ahead and substitute the \(x\)-values in. What happens? (When \(x = 3\), only one equation is true. Same for when \(x = 4\). But both equations are true when \(x = 10\).)
- So I hear you saying that you need to make sure that \((10, 80)\) is a solution to both equations. This is called finding the solution to a system of equations. (Marking)
- I’m also hearing that this point of intersection is the solution to our problem. We can find it in a table or graph, and check it with our equations. This intersection point is the point at which BOTH the \(x\)- and \(y\)-values are the same, so it gives us the number of classes for which the cost will be the same. (Recapping)
- How does the graph help us to determine which yoga company to choose when taking classes? (Challenging) (If you want to take less than 10 classes you should choose Stretch-It-Out, but after 10 classes, Flexible Yoga is cheaper.)
- How many solutions to this system of equations are there? (Challenging) (One.)
- Say more. How do you know? (Because two straight lines will only intersect one time.)
- We will be exploring this idea of one solution a bit more in the future. Meanwhile, I’m wondering if there’s a way that we can use the equations to algebraically determine when the cost will be the same. (Challenging)
| Application | Flexible Yoga changes their pricing. The studio reduces the base rate to $10 but charges $7 per class. How many classes would you now have to take for the two companies to cost the same amount? |
| Summary | What is a solution to a system of equations? How can you find it using a table, a graph, or an equation? |
| Quick Write | Explain which company offers the better deal for five classes. Mathematically support your answer. |

**Support for students who are English learners (EL):**
1. Bring in or display images or video of people in yoga classes or in yoga positions so that students identified as English learners associate the words in the problem with the images.
2. Continue to add to the English term/appropriate native language term for math terms relevant to the unit as is necessary.
3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can rehear and reprocess the day’s key ideas.
4. Continue the poster titled, *Strategies for Solving Systems of Equations*. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
Scuba Math

Part 1:
Serena and Dion took a scuba diving course while on vacation in Hawaii. On their first scuba diving trip, Serena swam toward the surface as Dion began his dive. The tables below represent their depth in feet with respect to time in seconds.

<table>
<thead>
<tr>
<th>Time (seconds)</th>
<th>Serena’s Depth (feet)</th>
<th>Time (seconds)</th>
<th>Dion’s Depth (feet)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-90</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>-80</td>
<td>20</td>
<td>-20</td>
</tr>
<tr>
<td>20</td>
<td>-70</td>
<td>40</td>
<td>-40</td>
</tr>
<tr>
<td>30</td>
<td>-60</td>
<td>60</td>
<td>-60</td>
</tr>
<tr>
<td>40</td>
<td>-50</td>
<td>80</td>
<td>-80</td>
</tr>
</tbody>
</table>

Assuming they continued swimming at the same rate, at what time were Dion and Serena at the same depth? Show all work and use equations to support your answer.
Part 2:
On the second day of their vacation, Serena and Dion went scuba diving again. Their depth in feet can be described by the following equations:

Serena: \( y = -50 + 2x \)

Dion: \( y = 1.5x \)

The following algebraic strategy can be used to determine the time at which they are at the same depth.

Step 1: \(-50 + 2x = 1.5x\)

Step 2:
\[-50 + 2x = 1.5x\]
\[1.5x - 1.5x\]

Step 3:
\[-50 + 0.5x = 0\]
\[+50 + 50\]

Step 4: \(0.5x = 50\)

Step 5: \(x = 100\)

1. Describe each step in the solution strategy and explain why each step makes sense algebraically.

2. Verify that \(x = 100\) represents a solution to the system of equations. Show all work and explain your reasoning.
Scuba Math

**Rationale for Lesson:** In the previous tasks, students made sense of the solution to a system of equations and then solved a system of equations given a context. In this task, students solve and make sense of the system from a table of values and a context. The exact solution cannot efficiently be determined using a graph or guess-and-check, so students are pressed to solve the solution algebraically. They are then pressed to analyze an algebraic solution, making sense of how the properties of equality are applied.

**Task 3: Scuba Math**

**Part 1:**
Serena and Dion took a scuba diving course while on vacation in Hawaii. On their first scuba diving trip, Serena swam toward the surface as Dion began his dive. The tables below represent their depth in feet with respect to time in seconds.

<table>
<thead>
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<td>-20</td>
</tr>
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<td>-70</td>
<td>40</td>
<td>-40</td>
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<tr>
<td>40</td>
<td>-50</td>
<td>80</td>
<td>-80</td>
</tr>
</tbody>
</table>

Assuming they continued swimming at the same rate, at what time were Dion and Serena at the same depth? Show all work and use equations to support your answer. **See student paper for complete task.**

**Common Core Content Standards**

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>8.EE.C.8a</th>
<th>8.EE.C.8b</th>
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<td></td>
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</table>

**Standards for Mathematical Practice**

<table>
<thead>
<tr>
<th>Standards for Mathematical Practice</th>
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<th>MP2</th>
<th>MP4</th>
<th>MP5</th>
<th>MP6</th>
<th>MP7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make sense of problems and persevere in solving them.</td>
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<td>Model with mathematics.</td>
<td>Use appropriate tools strategically.</td>
<td>Attend to precision.</td>
<td>Look for and make use of structure.</td>
<td></td>
</tr>
</tbody>
</table>
### Essential Understandings

*Greyed-out portions not addressed in task or lesson.

- The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.
- The solutions to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs \((x, y)\) that make the equation a true statement or satisfy the equation.
- The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.
- Because both the x-value and the y-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.

### Materials Needed

- Student reproducible task sheet
- Straight edge, graph paper, calculators (optional)
SET-UP PHASE
I’d like a student to read the task aloud while everybody else reads along silently. Without giving anything away about the solution, who can explain what this problem is all about? Has anybody ever been scuba diving? This picture shows the equipment that is needed. Why do you think a wetsuit is worn? Does anybody know how long divers can stay underwater? Is there a limit to how deep they can go? I’m going to give you 5-10 minutes to work independently on Part 1 before sharing ideas with the group. Graph paper, calculators, and rulers are available at your tables to use as needed.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

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</tr>
</thead>
<tbody>
<tr>
<td>Extends the tables of values.</td>
<td>Why did you extend the tables in this way? What is represented in each table?</td>
<td>Explain how you can determine the solution from this table. What does the solution represent?</td>
</tr>
<tr>
<td>Plots points and creates graphs.</td>
<td>What does the graph tell us about the problem? Explain what the intersection point represents in this problem situation.</td>
<td>Can you determine the exact solution from your graph? How can you confirm your answer using the table or equations?</td>
</tr>
<tr>
<td>Writes the equations and uses guess-and-check.</td>
<td>What do these equations represent in the problem situation?</td>
<td>What can you tell me about the y-values of the solution to the problem? Why? How can you use that fact and the equations to determine where Serena and Dion will meet?</td>
</tr>
</tbody>
</table>

Serena: \( y = -90 + 1x \)
Dion: \( y = -1x \)
Students then substitute values.
<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Writes the equations and solves algebraically using the properties of equality.</strong></td>
<td>Can you explain your solution strategy to me? Why did you set (-90 + 1x) equal to (-1x)? Why are the expressions still equal after each step?</td>
<td>What is the meaning of (x) in the context of the problem? How can you be sure that your solution for (x) is where Serena and Dion meet?</td>
</tr>
<tr>
<td>(-90 + 1x = -1x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(-90 = -2x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(45 = x)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Finishes early.</strong></td>
<td>Can you explain your solution strategy?</td>
<td>Can you explain to me how you know from looking at the equations (tables/graphs) that Dion and Serena will meet? How will this appear graphically (in an equation/table)?</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

- While circulating, I noticed several different strategies that groups were using to solve this problem. I noticed some groups using tables of values, some using a graph, and others using equations. Before we discuss the exact solution, who can explain what we’re trying to find here? *(Where Serena and Dion meet.)*
- Say more. Who can add on to this? *(Where they’re at the same depth.)*
- Okay, so looking at the table, I can see (pointing) that Serena and Dion are both at -80 feet. Is this a solution? *(Challenging)* *(No, both the time and depth have to be the same, not just the depth.)*
- Okay, so I’m hearing that the solution is the point at which the \((x, y)\) values, or the time and depth, are the same. *(Marking)*
- Since many people did the problem different ways, let’s explore how the solution will appear in the graph, table, and equation.

EU: The solutions to a linear equation in two variables can be represented graphically by a line consisting of all of the points represented by the ordered pairs \((x, y)\) that make the equation a true statement or satisfy the equation.

EU: The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

- We have previously discussed what the solution to a system of equations looks like in different representations. Today, let’s first look at a graphic solution. Group A, explain your graphs to us.
- Who can restate how you know which line represents Serena, and which line represents Dion? *(The one is going up and the other is going down.)*
- Who can say more? *(From the lines that are shown, we can determine where each person started, at the y-intercept, and how fast they were swimming.)*
- So if I look at just Serena’s line, how many solutions are there to the equation that represents this line? *(There are infinitely many, because there are infinitely many points on the line.)* What about Dion’s?
- Who can explain what it means to be a solution to the equation that represents each line? *(For a given time, you have the depth that matches the time.)*
- Who can add on? *(You have the depth that matches the time means that you can plug in or substitute the depth and matching time into the equation and make a true statement.)*
- I’m hearing that there are infinitely many solutions to each line and these solutions appear as points on the line. *(Marking)* All the points on the line, when substituted into the matching equation, make that equation a true statement. Another way to say that is that all the points on the line satisfy the equation.
- So then what is unique about this point of intersection right here? *(That’s a solution to both lines. The x- and y-values are the same.)*
- How do they know it’s a solution to both lines? *(The point is on both lines. The x and y values are the same. The value is \([45, -45]\).*

*continued on next page*
• So when you say that the x- and y-values are the same, do you mean both are 45? (Challenging) (No. They are both 45, yes, but the x- and y-values being the same means they are the same for each line.)
• Who understands what he just said and can say it in your own words? (I think I understand. Look over at Yoga from yesterday. The x and y for the point of intersection were the same for both lines; 10 classes, $80 so the point (10, 80). But 10 is not the same as 80.)
• Who agrees and can add on? What do we mean by the same for both lines? (Makes the equation for both lines true.)
• So we are saying that we are looking for a point that makes both equations true, satisfies both equations, simultaneously. Or at the same time. (Revoicing and Marking) We can also see how confusing it can get when we do not use the language of mathematics in our explanations. So, let’s try that question again. What is unique about this point of intersection? Let’s hear the answer from several of you. (Makes both equations true.) (Satisfies both equations.) (Simultaneously!) (Is the solution to each of the equations at the same time.)
• Who can explain what this point represents in this problem? (After 45 seconds, both Serena and Dion will be 45 feet below the surface.)
• This is important. We discussed that the x- and y-values will satisfy both equations at the same time and that this solution to the system appears graphically at the point of intersection. (Recapping)
• So can there be more than one solution to the system shown on this graph? (Challenging)

EU: Because both the x-value and the y-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.
• Who can explain how they made sense of this problem using equations?
• Who can tell us more about their equations $y = -90 + x$ and $y = -1x$? Explain how each of the equations represents this specific problem situation. (You can tell from the equation that Serena starts going up at -90 feet and for each 10-second interval, she goes up 10 feet, so her rate is 1 foot per second. Similarly, you can tell that Dion starts at the surface and swims down at 1 foot per second.)
• Who agrees or disagrees?
• Who can explain this in their own words using the table of values and/or the graph to help visualize their path?
• We already agreed that the point of intersection on the graph represents the solution, and this is (45, -45). How can we confirm this using the equations? (If you substitute 45 into both equations, you get -45 as your y-value.)
• Who can add on? (I knew the equations, and I could tell from the table of values that the solution was somewhere between 40 and 50, so I plugged in values until I found the value that gave the same output; this is the solution of 45 seconds and -45 feet.)
• This is an important point; you can use the equation to substitute the value and make sure each equation produces the same output for this value. (Marking)
• Do we need to use guess-and-check? Let’s hear from a group that algebraically determined the solution from the equations, using the properties of equality.
• Who can add on by explaining each step in their solution process? (They set the expressions equal to each other and then added 90 to both sides, added 1x to both sides, and then divided by 2.)
• Can you do that? Can you just set the expressions equal to each other like that? Why?  
(Challenging) (Yes, since both expressions are equal to y, then they must equal each other.)
• Who remembers the name of the property of equality that says since both expressions are equal to y, then they must equal each other? (The transitive property of equality.)
• (Student name), will you use the context to help us understand why we can set the expressions equal to each other? (At the point of intersection, the y-values are the same, and we are looking for that point.)
• Yes, that is an important point. Graphically, we can justify setting the expressions equal to one another because, at the intersection point, the y-values are equal. (Marking)
• What about contextually? Who can point to words in the context that suggest we can set the y-values equal to each other? (...at what time were Dion and Serena at the same depth.)
• Who can add on? Why do those words in the context suggest that we set the y-values equal to each other? (The depth is given by the y-values. The question says, “When are the depths the same?” That’s like saying, when are the y-values the same?)
• I have now heard three different ways to justify why we set 90 + x equal to -1x. We can claim that both are equal to y, and use the transitive property of equality. We can note that at the point of intersection, the y-values must be equal. We can reread the context and note that the context is asking when the depths or y-values are equal. (Recapping) These are all important justifications for our reasoning. Let’s add solving algebraically to our chart and also add these justifications. (Marking)
• Let’s now look at the solution to the equation. (Note: Remind the class what properties of equality we are using for each step.)
• Is this the only way to solve algebraically? (Challenging) Can anyone think of another way? (As long as you do the same thing to both sides, you can do it any number of ways.)
• So then why did they choose the numbers they chose? (To get x by itself.)
• Why do that? Why isolate x? Let’s see if we can explain algebraically, graphically, and contextually again. (x represents the time.) (We want the x-value at the point of intersection.) (And the context says, at what time! We have answered what time!)
• So I’m hearing that we can solve a system algebraically when the equations are in \( y = mx + b \) form by setting the expressions, the \( mx + b \) expressions, equal to each other and applying the properties of equality to isolate x. (Recapping) This method is called the substitution method for solving a system of equations. (Marking) Let’s label the method on our chart.
• How is this the same/different than other equations that we have encountered so far?

| Application | Suppose the paths of two divers are represented by the equations \( y = -50 + 1.5x \) and \( y = -1x \). Where will the divers meet? Show all work and explain your reasoning. |
| Summary | Describe what it means to be a solution to a system of equations. |
| Quick Write | Describe how the equations in the application problem represent the path of each diver. |
Support for students who are English learners (EL):
1. Bring in or display images or video of scuba divers so that students identified as English learners associate the words in the problem with the images.
2. Continue to add to the English term/appropriate native language term for math terms relevant to the unit as is necessary (here, “satisfy an equation,” “simultaneously,” and “substitution”).
3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can rehear and reprocess the day’s key ideas.
4. Continue the poster titled, *Strategies for Solving Systems of Equations*. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
Name_________________________________________________________

Income & Expenses

*Raphael is starting his own business making and selling customized bumper stickers.*

He makes an initial investment of $150 for equipment, and the materials for each bumper sticker cost $0.50. Raphael plans to sell the bumper stickers for $3.00 each. The equations below model Raphael’s income and expenses, where \( x \) represents the number of bumper stickers and \( y \) represents dollars.

\[
\text{Income: } y = 3x
\]
\[
\text{Expenses: } y = 150 + 0.50x
\]

1. a. What does the solution to the system of equations represent in terms of this problem situation?

b. Is \((65, 195)\) a solution to the system of equations? If so, use the equations to explain how you know it is a solution. If not, use the equations to determine the solution and then explain your reasoning.

c. Explain how you know that \((100, 300)\) is not a solution to the system of equations.
2. Julian is opening a competing bumper sticker business. His income and expenses are shown in the tables of values below.

<table>
<thead>
<tr>
<th>Stickers</th>
<th>Income ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>20</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>15</td>
<td>60</td>
</tr>
<tr>
<td>20</td>
<td>80</td>
</tr>
<tr>
<td>25</td>
<td>100</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Stickers</th>
<th>Expenses ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>75</td>
</tr>
<tr>
<td>30</td>
<td>150</td>
</tr>
<tr>
<td>55</td>
<td>225</td>
</tr>
<tr>
<td>85</td>
<td>315</td>
</tr>
<tr>
<td>95</td>
<td>345</td>
</tr>
</tbody>
</table>

For how many stickers will Julian’s income equal his expenses? Show all work and explain your reasoning. Use equations to support your answer.

3. What does it mean for a coordinate pair \((x, y)\) to be a solution to a system of equations?
Income & Expenses

**Rationale for Lesson:** In this task, students are asked to solidify their findings about a solution to a system of equations by analyzing an income and expenses situation. Students must find the break-even point from a table of values when the solution is not obvious. Therefore, they must write and use equations to find the solution.

**Task 4: Income & Expenses**
Raphael is starting his own business making and selling customized bumper stickers. He makes an initial investment of $150 for equipment, and the materials for each bumper sticker cost $0.50. Raphael plans to sell the bumper stickers for $3.00 each. The equations below model Raphael’s income and expenses, where \( x \) represents the number of bumper stickers and \( y \) represents dollars.

- **Income:** \( y = 3x \)
- **Expenses:** \( y = 150 + 0.50x \)

1. a. What does the solution to the system of equations represent in terms of this problem situation?
   b. Is (65, 195) a solution to the system of equations? If so, use the equations to explain how you know it is a solution. If not, use the equations to determine the solution and then explain your reasoning.
   c. Explain how you know that (100, 300) is not a solution to the system of equations.

*See student paper for complete task.*

**Common Core Content Standards**

| 8.EE.C.8a | Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. |
| 8.EE.C.8b | Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, \( 3x + 2y = 5 \) and \( 3x + 2y = 6 \) have no solution because \( 3x + 2y \) cannot simultaneously be 5 and 6. |

**Standards for Mathematical Practice**

| MP1 | Make sense of problems and persevere in solving them. |
| MP2 | Reason abstractly and quantitatively. |
| MP4 | Model with mathematics. |
| MP5 | Use appropriate tools strategically. |
| MP6 | Attend to precision. |
| MP7 | Look for and make use of structure. |
### Essential Understandings

- The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.
- The solution to a system of two or more linear equations can be represented algebraically, graphically, in a table, and in context.
- Because both the x-value and the y-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.

### Materials Needed

- Student reproducible task sheet
- Straight edge, graph paper, calculators (optional)
SET-UP PHASE
I’d like a student to read the task aloud while everybody else reads along silently. Has anyone ever tried to run their own business (lawn mowing, maybe a lemonade stand when you were little, etc.)? What were your expenses before you got started? Why is this important to you? How will you make a profit? You may use any of our solution strategies to support your thinking in this task, but let’s also be sure to find the solution using the equations as well, like we did in the Scuba Math task. Graph paper, calculators, and rulers are available at your tables to use as needed.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>For question 2:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plots points and</td>
<td>What does the graph tell us about the problem? Explain what the intersection point represents in this problem situation.</td>
<td>How can you write equations from the information given?</td>
</tr>
<tr>
<td>creates graphs.</td>
<td></td>
<td>How can you confirm your answer using the equations?</td>
</tr>
<tr>
<td><strong>Writes the equations and uses guess-and-check.</strong></td>
<td>What do these equations represent in the problem situation? How did you choose a reasonable guess for finding the solution?</td>
<td>How do you know that your x-value is a solution to the problem? How can you use what we discussed yesterday to find the solution using your equations?</td>
</tr>
<tr>
<td>Income: ( y = 4x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expenses: ( y = 3x + 60 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Students then substitute values.</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Finishes early.</strong></td>
<td>Can you explain your solution strategy to me? How will you find your income after selling this many bumper stickers? What will your profit be at this point?</td>
<td>When will Julian begin to make a profit? If Julian sells 100 bumper stickers, what will his profit be?</td>
</tr>
<tr>
<td>4x = 3x + 60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1x = 60</td>
<td></td>
<td></td>
</tr>
<tr>
<td>x = 60</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

- In Part b, you were asked to verify if \((65, 195)\) was a solution to the system of equations. Was it a solution to \(y = 3x\)? (Yes.) So it’s a solution to the system then, right? **(Challenging)**
  - (No, because it is not a solution to the other equation.) How do you know? (When you plug in 65 bumper stickers for \(x\), you will get $182.50, not $195.)
- (Student name), will you restate for us what it means to be the solution to a system of equations?
- Who agrees? Disagrees?
- Okay, so I’m hearing that the solution is the point at which the \((x, y)\) values, or the number of bumper stickers and money, satisfy both equations, make both equations true. **(Marking)**
- In this context, why do we care about the point where the number of bumper stickers and expenses and income are the same for both income and expenses? (It is the point where the amount of money spent equals the amount of money made.)
- Say more. Why do you want to know about this point in a business? (So you can start making money. Now he will have a profit.)
- Since every business wants to make money, this is an important point. It is called the break-even point.
- Let's look at how you represented the next problem situation since you were given a table of values for income and expenses.
EU: The solution to a system of two or more linear equations can be represented algebraically, graphically, in a table, and in context.

EU: Because both the x-value and the y-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.

- When we have used tables in the past, we’ve been able to find the solution. Why is that more difficult here? *(There is not a point shown in these table values where x and y are the same for both.)*
- Both? Say more. *(For both the number of stickers and the income.)*
- *(Student name), you are disagreeing? *(For both the income and expenses.)*
- If there is not a point shown in the table, do you think that means there is no solution to the problem? *(Challenging) We know there is. The graphs intersect.)*
- Why don’t we see that in the table, then? *(There are several x-values skipped in each table so it could be there.)*
- Suppose we did not create a graph. How can we use the rate of change for income and expenses to know if there will be a point of intersection? *(Challenging) Yes, if the slope of the two lines are different, the lines will eventually intersect somewhere.)*
- Let’s look at this group’s graph and how they found the solution. How do they know it’s a solution to the problem? *(The point is on both lines. The x- and y-values are the same. The point is [60, 240].)*
- Are the x- and y-values the same if x = 60 and y = 240? How can we better express how we know the intersection point is a solution to the problem? *(The intersection point is on the income line and the expenses line.)*
- Who can say more? *(It satisfies both income and expense equations.) (The x is the same for income and expense, and the y is the same for income and expense.) (It satisfies both income and expense simultaneously.)*
- So let me mark a clearer use of mathematical language in these last responses. *(Marking)*

Let me challenge you all in our next few lessons to try for this clearer use of mathematical language. Will you all take a moment to stop and jot these clearer ways of justifying your reasoning? I will have the students repeat what they just said.

- How can we represent this problem with equations? What are the equations and how did you get them? *(By finding the slope and y-intercept so y = 4x and y = 3x + 60.)*
- How can we use the equations to find the solution to this system? *(Since we want the expressions 3x + 60 and 4x to equal the same thing, we can set them equal and solve 4x = 3x + 60.)*
- Why? Look at our chart. Who can give me the contextual reason?
- Who can give me the algebraic reason?
- Who can give me the graphical reason?
- How do you solve this equation?
- So if x = 60, what does this mean for the problem situation? *(x = 60 means that we need to sell 60 bumper stickers for income and expenses to be equal.)*
- How do we find y? *(Plug 60 into one of the equations.) Can someone show us what this substitution looks like?*
- How can we confirm that we have found the solution to the system? *(Plug 60 into the other equation to make sure 240 is the amount of money.)*

*continued on next page*
• So what is the break-even point? (60, 240.) What is the profit at this point? (0, because income and expenses are equal.)
• Why do you think it is beneficial to know an algebraic strategy for solving a system of equations? (It isn’t always practical or accurate to make a graph by hand.) (You may not find the exact value on the graph.) (You may not find the value at all in the table.)
• So let’s sum this up. What I heard everybody say was that we can solve the system algebraically to determine the \( x \)-value and that this \( x \)-value can be used to determine the \( y \)-value. The algebraic strategy gives us the common \( x \)-value and \( y \)-value for both equations. This is the solution to the system. We’ve also seen it as the point where the lines intersect on a graph and a common point in the table. (Recapping)

### Application
Find the break-even point for a t-shirt selling business that has income and expenses modeled by the following equations:

- Income: \( y = 15.99x \)
- Expenses: \( y = 250 + 5.5x \)

### Summary
How can we use equations to find the solution to a system of equations?

### Quick Write
Explain how to use the properties of equality to solve a system of equations.

### Support for students who are English learners (EL):
1. Discuss situations involving income and expense, so that students identified as English learners associate the words in the problem with the situations.
2. Continue to add to the English term/appropriate native language term for math terms relevant to the unit as is necessary.
3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can reheat and reprocess the day’s key ideas.
4. Continue the poster titled, **Strategies for Solving Systems of Equations**. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
To Meet or Not to Meet?

Blaire thinks that when you graph two lines, they will have one point of intersection. Dan thinks that the lines can also be parallel or that they can coincide. While graphing equations on the coordinate plane, they realize that they are both correct, but they are not sure exactly why or when. Help them figure out the answer.

a. Predict the conditions under which the equations of two lines will meet in one point, will be parallel, or will coincide. Use drawings, equations, and words to write about the thinking behind your predictions.

Exploration

b. Using graph paper, graph two lines that meet in exactly one point. Find the equation of each line. Share your graphs and equations with a partner, then with another pair. What do you notice? Using what you have discovered, revise or add on to your prediction.
c. The graph below shows two parallel lines. Determine the equation of each line. Determine the equations of another line parallel to the two below. Share your equation with a partner, then with another pair. What do you notice? Using what you have discovered, revise or add on to your prediction.

\[
\begin{align*}
2x + 4y &= -10 \\
y &= \left(\frac{1}{2} x\right) - 2.5
\end{align*}
\]

Write about what you notice about the graphs of the two lines. Discuss your observations with a partner, then with another pair. Using what you have discovered, revise or add on to your prediction.

e. If you are given two equations with the same independent and dependent variable, how can you decide (without graphing) whether the equations will coincide, be parallel, or meet in exactly one point when graphed on the same axes?
To Meet or Not to Meet?

Rationale for Lesson: Students explore the conditions under which pairs of lines have one solution, no solutions, or infinitely many solutions.

Task 5: To Meet or Not to Meet?
Blaire thinks that when you graph two lines they will always have one point of intersection. Dan thinks that the lines could also be parallel or that they could coincide. While graphing equations on the coordinate plane, they realize that they are both correct, but they are not sure exactly why or when. Help them figure out the answer.
a. Predict the conditions under which the equations of two lines will meet in one point, will be parallel, or will coincide. Use drawings, equations, and words to write about the thinking behind your predictions.

Exploration
b. Using graph paper, graph two lines that meet in exactly one point. Find the equation of each line. Share your graphs and equations with a partner, then with another pair. What do you notice? Using what you have discovered, revise or add on to your prediction.

See student paper for complete task.

Common Core Content Standards

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>8.EE.C.8a</th>
<th>8.EE.C.8b</th>
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<tbody>
<tr>
<td></td>
<td>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</td>
<td>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</td>
</tr>
</tbody>
</table>

Standards for Mathematical Practice

| Standards for Mathematical Practice | MP1 | Make sense of problems and persevere in solving them. | MP2 | Reason abstractly and quantitatively. | MP3 | Construct viable arguments and critique the reasoning of others. | MP4 | Model with mathematics. | MP5 | Use appropriate tools strategically. | MP6 | Attend to precision. | MP7 | Look for and make use of structure. | MP8 | Look for and express regularity in repeated reasoning. |

Essential Understandings

- Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution.
- Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions.
- Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions.

Materials Needed

- Student reproducible task sheet
- Straight edge, grid paper/additional lined paper, graphing calculators (optional)
### SET-UP PHASE
I’d like a student to read the task aloud while everybody else reads along silently. Who, in their own words, can describe what this lesson is about? What does it mean to make a prediction? What does it mean to “predict the conditions under which the equations of two lines will meet in one point, will be parallel, or will coincide”? Who can draw a sketch of two lines meeting in one point? Two parallel lines? Two coinciding lines? You have additional graph paper, graphing calculators, rulers, and paper available to each group. As you gather evidence and explore each situation, use the materials as needed.

### EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Can’t get started.</td>
<td>What are you being asked to explore? On the grid paper can you show me the graphs of two lines that meet at one point?</td>
<td>How can you determine the equations of the lines? What do you notice about the equations of the lines?</td>
</tr>
<tr>
<td><strong>For part c:</strong> Makes sense of the problem using a table of values. (May use two separate tables.)</td>
<td>How/why did you set up the table of values? What information is evident in the table of values?</td>
<td>What patterns do you notice? How will the patterns help you sketch a third line that does not intersect either of the lines shown?</td>
</tr>
<tr>
<td>x</td>
<td>y&lt;sub&gt;1&lt;/sub&gt;</td>
<td>y&lt;sub&gt;2&lt;/sub&gt;</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>6</td>
<td>-5</td>
<td>5</td>
</tr>
<tr>
<td><strong>For part c:</strong> Sketches the line but is unable to make sense of the situation algebraically.</td>
<td>Tell me about your graph. How did you know to sketch your line in this way?</td>
<td>What information do you need to determine the equation of a line? What can you do to gather that information?</td>
</tr>
<tr>
<td><strong>Determines the equations algebraically for part c, conjectures that slopes of parallel lines must be the same.</strong></td>
<td>What led you to the conclusion that parallel lines have the same slopes? What does this mean in terms of their solutions?</td>
<td>Is it possible for pairs of lines to have the same slope but have solutions in common? Explain.</td>
</tr>
<tr>
<td><strong>Finishes early.</strong></td>
<td>Can you explain the various conditions you studied and what you learned about the number of solutions for pairs of lines?</td>
<td>Can you write the equations for pairs of lines that have no solution in common, one solution in common, and infinitely many in common? Explain.</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

**EU:** Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution.

- Who can tell me about the original predictions that your groups made? (Answers will vary.)
- What was the significance of making the prediction first and then doing the exploration? (We first make a conjecture about what we think is true and why, and then search for evidence.)
- Who can add on to this? (It’s how scientists work. You have a theory and then you test it out.)
- Before we get into the problems at hand, what does it mean to say that lines intersect? (It means they cross.)
- Say more. What does this intersection point represent? (That’s the solution, the value \([x, y]\) that satisfies both of them.)
- Who can restate this in their own words?
- Who can share how their group made sense of lines that intersect once? (We drew several lines that intersect only once. We split up the work and each determined equations. We saw no patterns.)
- Say more. (We didn’t notice anything similar about the equations.)
- Who can say more about there being nothing similar? (The slopes and y-intercepts were all different.)
- Does that mean the slopes and y-intercepts must be different? (Challenging) Is it possible to sketch two lines that intersect at the y-intercept but have different slopes?
- How will the equations of these lines be the same/different? (The \(m\) values will be different but the \(b\) values will be the same.)
- Who can restate what was just said in his/her own words? What is meant by \(m\) values and \(b\) values? (When written in \(y = mx + b\) form, \(m\) is the slope and \(b\) is the y-intercept.)
- Who can give me the equations of two distinct lines that intersect at \((0, 3)\)? (\(y = 2x + 3\) and \(y = 5x + 3\).)
- Who can explain how they know these lines will intersect only once? (They will continue increasing or decreasing at different rates after this one point of intersection.)
- We see many examples of lines that intersect at points other than the y-intercept. Without necessarily determining the exact equations, what must be true of these pairs of lines? (We discussed how their y-intercepts may be the same or different, but the slopes are all different.)
- This is an important point. The slopes of each pair of lines that intersect are different. (Marking) Who can explain how they knew this? (They’re going up or down at different rates until they intersect.)
- So if two lines intersect, then their slopes must be different. Does it follow then if two lines have different slopes then they must intersect?
- It looks like we are in agreement that if two lines intersect then their slopes are different and if the slopes of two lines are different, then they intersect. We can see this on the graphs shown that the rates of change are different. (Recapping)
EU: Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions.

- We discussed the fact that intersecting lines must have different slopes. Who can explain what can happen when two lines have the same slope? (Challenging) /Lines that have the same slope are parallel./
- Did your work on Part c verify that parallel lines have the same slope? What does it mean for lines to be parallel? (They never intersect.)
- How do we know that the lines in Part c will never intersect, even after the lines go beyond this page? (We found the slope of both lines. We used that slope to graph a third line. Because the slopes are the same, the lines keep decreasing at the same rate.)
- Who can add on or say more? (We made a bunch of slope triangles. When you do that, you can see that going down by the same amount means the lines stay the same amount apart.)
- One group used a table to examine these lines. Does your table verify what was just said? (Yes, look here. At 0 and 3 and 6 and 9, the y-values are always 10 apart. So if they keep changing at the same rate, they will keep staying 10 apart. They can’t meet if that keeps happening.)
- So both the slope triangles and the table are convincing us that parallel lines have the same slope. (Marking)
- How many solutions does the system of equations with parallel lines have? Explain. (They have no solutions.)
- What does it mean for a system to have no solutions? (They have different outputs for any input value.)
- Who can add on? (Parallel lines never intersect, so there is no point of intersection and no solution.)
- Anyone else? (There is no point where parallel lines have a common x- and y-value simultaneously.)
- So parallel lines increase at the same rate and never intersect. They have no solutions because the y-values are different for any given x-value. (Marking)
- Who can show other parallel lines that their group drew?
- How do we know that the lines are parallel?
- How does this parallel relationship show up in their equations? (The slope is the same. In this case the slopes are $\frac{5}{3}$.)
- Who can give several different examples of lines and their equations that will be parallel to these two lines? Must we graph them, or can we know simply from their equations? Explain.
- So do all lines parallel to these lines have to have a slope of $\frac{5}{3}$? (Challenging) Explain. (Yes, because in order to be above or below the lines and never touch it must keep going down 5 and over 3 in this same pattern.)
- From the discussion it sounds like we’re all in agreement that parallel lines have the same slope and in the equation this appears as the same m-value when the equations are written in the form, $y = mx + b$. (Recapping)
EU: Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions.

- So what happens when the slopes are the same and the y-intercepts are also the same? (The lines are directly on top of each other.)
- Do you agree/disagree?
- Who can show on the graph why this is true?
- What does this mean in terms of the solution to the system of equations? (They have infinitely many solutions.)
- Who can add on? What is meant by “infinitely many solutions”?
- Do the equations have to look the same? For example, \( y = 2x + 5 \) and \( y = 2x + 5 \) are obviously the same line. Can a system of equations look different and still have infinitely many solutions?
- I heard some groups say yes. Explain. (When we graphed the two equations in Part d, they are the same line.)
- Can they be the same line? Their equations look completely different. (Yes. When you plug in any x-value you get the same y-value.)
- How do you know? It’s impossible to plug in every value, right? How many values do you need to substitute to be sure?
- Did anybody use a method other than substituting values? (If you solve the first equation for \( y \), you get \( y = (\frac{3}{2}x) - 2.5 \))
- So, when you solve both equations for \( y \) using the properties of equality, you realize that the equations are equivalent. (Marking)
- Can you manipulate the equations in other ways to show they’re equivalent? (Challenging)
- When the equations are equivalent, even though they may not originally look the same, how many solutions are there to the system of equations? (Infinitely many.)
- Someone say why there are infinitely many solutions. (Well, there is really only one line, and we said a long time ago that the equation of one line has infinitely many solutions.)
- We’ve looked at three different possibilities for pairs of lines: either they intersect once, infinitely many times, or they never intersect. If the slopes are the same but the y-intercept differs, they are parallel; if the slopes are different, they intersect once; and if the equations are equivalent, they coincide. (Recapping)
- Let’s add this to our chart, and add one more critical piece of information there. Someone recap for me. If the two lines have different slopes, how many solutions will we be expecting? If the two lines are parallel, how many solutions will we be expecting? If the two lines are equivalent, how many solutions will we be expecting?

<table>
<thead>
<tr>
<th>Application</th>
<th>Determine whether each pair of equations has one solution, no solution, or infinitely many solutions. Explain your reasoning.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( y = 2x + 3 ) and (-2y = 4x + 6)</td>
<td></td>
</tr>
<tr>
<td>2. ( y = x + 2 ) and ( y = x + 7)</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Summary</th>
<th>See recapping above.</th>
</tr>
</thead>
</table>

| Quick Write | No Quick Write for students. |
Support for students who are English learners (EL):

1. Allow students to sketch the three situations and leave the sketches up during the Explore Phase, so that students identified as English learners associate the words in the problem with the situations.

2. Continue to add to the English term/appropriate native language term for math terms relevant to the unit as is necessary (here, “parallel” and “coincide”).

3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can rehear and reprocess the day’s key ideas. Ask them to point to parallel lines when they are discussed, and to the point of intersection as it is discussed. Have them trace with their finger the lines that coincide as they are discussed.

4. Continue the poster titled, Strategies for Solving Systems of Equations. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
Savings Plans

*Tiana and Katie are creating savings plans for college. They plan on depositing the same amount of money into their accounts each week. Tiana’s savings plan is shown in the table of values below.*

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>Tiana’s Savings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>37</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
</tr>
<tr>
<td>7</td>
<td>67</td>
</tr>
<tr>
<td>9</td>
<td>82</td>
</tr>
<tr>
<td>11</td>
<td>97</td>
</tr>
</tbody>
</table>

1. Create a savings plan for Katie that meets the following conditions:
   - She saves money at a constant weekly rate.
   - She has more money than Tiana in her account at 0 weeks, but ends up with less money than Tiana after 15 weeks.

2. Katie and Tiana share their savings plans with one another. Katie claims that she and Tiana will have the same amount of money in their account at the same number of weeks several times during the next year. Is she correct? Justify your answer using equations and/or words, graphs, and tables.
Savings Plans

Rationale for Lesson: In this task, students move from looking for the solution to a system of linear equations to considering the possible numbers of solutions. In particular, this lesson focuses on the conditions under which a system of linear equations will have exactly one solution. After this lesson, students will analyze the conditions under which a system has zero or infinitely many solutions.

Task 6: Savings Plans
Tiana and Katie are creating savings plans for college. They plan on depositing the same amount of money into their accounts each week. Tiana’s savings plan is shown in the table of values below.

1. Create a savings plan for Katie that meets the following conditions:
   • She saves money at a constant weekly rate.
   • She has more money than Tiana in her account at 0 weeks, but ends up with less money than Tiana after 15 weeks.

2. Katie and Tiana share their savings plans with one another. Katie claims that she and Tiana will have the same amount of money in their account at the same number of weeks several times during the next year. Is she correct? Justify your answer using equations and/or words, graphs, and tables.

See student paper for table.

<table>
<thead>
<tr>
<th>Common Core Content Standards</th>
<th>8.EE.C.8a</th>
<th>Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>8.EE.C.8b</td>
<td>Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>Standards for Mathematical Practice</th>
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<tbody>
<tr>
<td>MP1  Make sense of problems and persevere in solving them.</td>
</tr>
<tr>
<td>MP2  Reason abstractly and quantitatively.</td>
</tr>
<tr>
<td>MP4  Model with mathematics.</td>
</tr>
<tr>
<td>MP5  Use appropriate tools strategically.</td>
</tr>
<tr>
<td>MP6  Attend to precision.</td>
</tr>
<tr>
<td>MP7  Look for and make use of structure.</td>
</tr>
</tbody>
</table>
| Essential Understandings | \[ \text{The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs } (x, y) \text{ that make both equations true statements or satisfy the equations simultaneously.} \\
\text{Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution.} \\
\text{Because both the } x\text{-value and the } y\text{-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.} |
|---|---|
| Materials Needed | \[ \text{Student reproducible task sheet} \\
\text{Straight edge, grid paper/additional lined paper, calculators (optional)} \]
SET-UP PHASE

I’d like a student to read the task aloud while everybody else reads along silently. As you are reading, highlight key information that will be useful in the problem. Who can say, in their own words, what this problem is about? How many people have started thinking about college? College can be expensive, so saving early is usually a good idea. Without giving away a solution, who can describe Tiana’s plan and what you know will be true about Katie’s plan? I’ll give you about 5-10 minutes to work independently on some ideas for Katie’s plan before you work in groups. By the way, there may be many different possible savings plans, so bring your own ideas for the group to discuss. It’s not necessary for every member of a group to have the same savings plan, but I want you to look for similarities across the plans.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

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<tr>
<th>Possible Student Pathways</th>
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<th>Advancing Questions</th>
</tr>
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<tbody>
<tr>
<td><strong>Group can’t get started.</strong></td>
<td>What can you tell me about Tiana’s plan? What must be true about Katie’s?</td>
<td>If Katie has more money in her account at zero weeks, what might a possible starting value for her be? Explain. Write this information down in a table.</td>
</tr>
<tr>
<td>Creates and extends the table of values (may be two separate tables).</td>
<td>Tell me about Tiana’s and Katie’s plans on the table. What patterns do you notice in each girl’s savings plan?</td>
<td>How can you verify from your table when/how many times the girls will have the same amount of money at the same number of weeks? How will this appear on a graph?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time (weeks)</th>
<th>Tiana’s Savings ($)</th>
<th>Katie’s Savings ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>14.50</td>
<td>20.00</td>
</tr>
<tr>
<td>1</td>
<td>22.00</td>
<td>25.00</td>
</tr>
<tr>
<td>2</td>
<td>29.50</td>
<td>30.00</td>
</tr>
<tr>
<td>3</td>
<td>37.00</td>
<td>35.00</td>
</tr>
<tr>
<td>4</td>
<td>44.50</td>
<td>40.00</td>
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<tr>
<td>5</td>
<td>52.00</td>
<td>45.00</td>
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<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>15</td>
<td>137.00</td>
<td>95.00</td>
</tr>
</tbody>
</table>
### Possible Student Pathways

<table>
<thead>
<tr>
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<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plots points and creates a graph.</td>
<td>What does the graph tell us about Katie’s and Tiana’s savings plans?</td>
<td>What does your graph suggest about the answer to question 2? Explain.</td>
</tr>
<tr>
<td>Uses equations.</td>
<td>What do these equations represent? Why do they meet the criteria mentioned in the problem? <em>(That Katie starts with more than Tiana, but will end up with less money over time.)</em></td>
<td>How do your equations help to answer question 2? How can you verify this relationship graphically?</td>
</tr>
<tr>
<td>Finishes early.</td>
<td>Can you explain your solution strategy?</td>
<td>Can you create a savings plan for Katie such that the graphs of their plans will never intersect? How will this relationship appear in the table and equation?</td>
</tr>
</tbody>
</table>
SHARE, DISCUSS, AND ANALYZE PHASE

EU: The solution(s) to a system of two linear equations in two variables is the ordered pair or pairs \((x, y)\) that make both equations true statements or satisfy the equations simultaneously.

- I noticed while circulating that several groups represented Katie’s savings plan first with a table of values. Who can share their solution strategy?
- Can another group explain what was just said, including why this savings plan meets the criteria that was given?
- What does the 0 in the table represent?
- What is each girl’s savings rate? How does this appear in the tables of values? *(The rate is the change from week to week.)* *(The number of weeks and the savings are going up by the same amount each week.)*
- That is how the rate appears in the table. Now, who can add on what the rate actually is and how you determined it?
- You noticed that the rate is determined by looking at the ratio of the difference between one amount and the next in the table. *(Marking)*
- How do the two girls’ savings plans compare with each other?
- Who can summarize in their own words what was just said, explaining why the starting point and rate of change are significant? *(Katie’s starting point is greater at first, but because Tiana’s rate of change is greater, she will catch up to Katie and pass her.)*
- This is important. I heard that she catches up and passes Katie. *(Marking)* Why does this happen? *(It happens because Tiana starts with less but since her rate is more, it isn’t long before she makes up for the fact that she started with less.)*
- Who can add on? What does she mean when she says it isn’t long before she “makes up for starting with less”? *(It means that since she has a greater rate, she is saving more each week, and she can quickly make up for the difference that they had to start.)*
- What does it mean to be a solution in this problem situation? *(The solution is the number of weeks when both girls will have saved the same amount. This is the number of weeks when we have made up for the difference between their starting amounts.)* *(The solution is the point on the graph or in the table where both plans have the same number of weeks and the same amount of savings.)*
- What other groups used a table to show Katie’s plan? Let’s take a look at all these tables. Do all the tables show the same plan for Katie?
- Then what IS the same? *(The pattern.)*
- Say more about the pattern. *(The constant rate of change, the ratio.)* *(They all show Tiana catching up and passing Katie.)* *(They don’t all show the place where the number of weeks is the same and the savings is the same, though.)*
- How many solutions appear in each of the tables of values? How do you know? *(One solution, because right here the one value is greater and the other is less and then they switch. So they will be the same at only one point.)*
- What about the table that did not show the place where the number of weeks is the same and the savings is the same? *(Well, they just don’t show it. But, since we still see the switch, we know it’s there.)*
- So I’m hearing you say that the solution appears in a table when the \(x\)- and \(y\)-values are the same for both plans and that this is occurring because the rates are different and the one girl “catches up” to the other after beginning with a different value. *(Recapping)* Can there ever be another solution? How do you know? *(No, because if you extend the lines they will not cross again.)*
• This is true and you are on the right track, but it is not the reason why there will not be another solution. Who can add on? Can someone give us the reason why there will not be another solution? (There will not be another solution because once Tiana has saved more than Katie; her amount keeps increasing at a greater rate so they will not be able to have the same amount in their savings accounts.)

• Who can add on? (After they switch, Tiana adds more and more money than Katie, so her plan will always have more money after that one time in the table.)

• Okay, we know it is always good to explore a problem using another representation, just to check our thinking. Let’s move on to a graph.

EU: Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution.

• This group showed the comparison of the amounts in a graph. Does the graph show the same information as the table? Why/why not?

• How do we know that each girl’s savings plan can be represented by a linear graph? (Each girl saves the same amount over time.)

• Will someone say more about what she means by “over time?” (They each have a constant rate of increase. So, as the weeks pass (over time), the constant rate shows up graphically as a line.)

• What is significant about the point of intersection? (This is where the girls have the same amount of money at the same number of weeks. This is the solution to the equations.)

• I’m hearing the point of intersection is the solution to this system of equations. (Revoicing and Marking)

• How do the x- and y-values compare at this point of intersection? (They are the same for both plans.)

• Say more; be more precise. (At x weeks, they both earned this amount.) (Note to teacher: Press students to identify specific amounts.)

• Who else used a graph? Let’s look at them all. Again, do all the graphs show the same plan for Katie? Then what IS the same?

• In this representation of the problem situation, a graph, can the girls have the same amount of money at two different times? Explain. (No, because the lines will never intersect again. Once one student has more in her account, then she will constantly gain even more because of her greater rate of increase.)

• How can you be sure that the lines will never intersect again? I’d like someone to demonstrate why this is true graphically. Specifically, what is true about the y-intercepts and the slope that guarantees the two girls will only ever have the same amount of money once? (Just like in the table of values, we can see that the starting point, which here is the y-intercept, is less for one of the graphs, but then its rate of change is greater so its line is steeper. That means it catches up and will intersect the other line and then keep going up at the steeper rate.)

• I’m hearing that if the slopes and y-intercepts are different, then the lines will intersect; however, once they intersect they will not intersect again. This is true and an important point that we need to note. (Recapping and Marking)

• Additionally, I am hearing you say that both the table and the graphic solution strategy to this system indicate clearly that there can be only one solution to the system. The table verifies the graph or the graph verifies the table on the issue of number of solutions. (Recapping)
**EU:** Because both the $x$-value and the $y$-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.

- Let’s look at the algebraic solution and see what it can add to the issue of number of solutions.
- I noticed several different equations to describe Katie’s plan while I was circulating, but I’d like to start with an explanation from Group D; they constructed the following equations to model the situation. Tiana: $y = 14.50 + 7.50x$ Katie: $y = 25 + 4x$
- What do the equations represent in this problem situation?
- How many solutions exist for each equation? *(There are infinitely many solutions because substituting an $x$-value results in a unique output and you can substitute any number of values.)*
- I’m hearing that each equation has infinitely many solutions. *(Marking)* We have heard that many times before! Now let’s talk about the system of equations.
- Who understands why this group wrote $14.50 + 7.50x = 25 + 4x$ and can explain it in your own words? Let’s look to our chart and explain why, both graphically and algebraically.
- Who can explain this group’s algebraic solution to the problem?

$14.50 + 7.50x = 25 + 4x$

$3.50x = 10.50$

$x = 3$

- *(They used the properties of equality to get $x$ by itself)* *(Student name)*, what does $x = 3$ mean in the context of this problem? Do we have the solution to the problem? *(Yes, it means 3 weeks. At 3 weeks, the savings are the same.)*
- How do we know that she is correct? *(If you substitute $x = 3$ into each equation, you get the same output, which is 37.)*
- Does that mean that $x = 3$ is a solution to this system? *(It’s the $x$-value, yes.)*
- So then what is the solution to the system of equations? And how do we know it is the solution for sure? *(The solution is (3 weeks, $37$). When you substitute (3, 37) into both equations, you get true statements.)*
- Several other groups also chose algebraic plans for Katie, though their plans were different than Group D’s. Someone from those groups explain how your plans were the same and how they differed. *(We had different equations, so, when we solved them, different solutions to the problem.)*
- Did you also have different solutions to the system? *(Yes.)*
- We have heard the differences. What are the similarities? *(We also found only one solution to the problem and the system.)*
- Who can use what we discussed with the graph and tables to explain why there is only one unique algebraic solution to the system? *(The solution to the system is the point at which the lines intersect. It’s the one $(x, y)$ coordinate that is the same for both equations. This can be seen when you solve the equation.)*
- I’m hearing that the algebraic solution supports what we saw in the table of values and in the graphs in that there is one unique solution $(x, y)$ to this system. I’m hearing additionally that, although each equation has infinitely many solutions, the system of linear equations with different rates of change and different $y$-intercepts will have exactly one $(x, y)$ coordinate that makes both equations true. *(Recapping)*
In this instance, the solution is \((3, 37)\). We verified this algebraically and saw how the graphs with one intersection point and the table of values with one common solution support this algebraic solution. **(Recapping and Marking)**

- Who can sum up in their own words the main ideas that we discussed today? How do you know when a system will have exactly one solution? Reference a graph, equation, and table in your explanation.
- Recall what we did in the To Meet or Not to Meet? exploration. How can you change the rate of change of Katie’s savings plan so that there will be no solutions to this problem? **(Challenging)**

### Application
Represent Katie’s savings plan with a different equation that meets the criteria.

### Summary
(See summary question at the end of the lesson.) Explain how you can determine from a graph and an equation whether two equations will intersect in exactly one point.

### Quick Write
No Quick Write for students.

### Support for students who are English learners (EL):
1. Allow students to discuss what it means to have a savings plan. Also allow students to see other tables, graphs, and equations so they can see the similarities and differences in the plans. You may want to use a gallery walk to examine the different plans, and discuss the similarities and differences in the tables, graphs, and equations before the Share, Discuss, and Analyze Phase begins. Such a discussion can serve as a review of slope and intercepts for all students.
2. Continue to add to the English term/appropriate native language term for math terms relevant to the unit as is necessary.
3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can reheat and reprocess the day’s key ideas. Ask them to point to parallel lines when they are discussed, and to the point of intersection as it is discussed. Have them trace with their finger the lines that coincide as they are discussed.
4. Continue the poster titled, **Strategies for Solving Systems of Equations**. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
Pick Your Programs

Many cable television companies charge a flat rate for a cable TV package. As an alternative, some cable companies are starting to offer packages where customers pay a flat rate for access plus a fee per channel. Two of these companies are described below.

- TV Party charges a $40.00 flat rate plus $1.50 per channel.
- Cable Club charges a flat rate of $20.00 plus $3.00 per channel.

1. Alicia argues that, since the cost per channel for Cable Club is twice the cost per channel of TV Party, but the flat rate is half of TV Party’s, then the companies will always cost the same amount for any number of channels. Do you agree or disagree with Alicia? Justify your reasoning using tables, equations, and graphs.

2. Create a cable plan with a flat rate and a cost per channel that is always less expensive than TV Party and Cable Club. Explain how you know that your plan will always be less expensive.

3. Is it possible to create a cable plan that is less expensive than TV Party for any number of channels, but more expensive than Cable Club for any number of channels? Use words and equations to justify your answer.
## Pick Your Programs

**Rationale for Lesson:** In the previous task, students considered the conditions under which a system of linear equations has one solution. In this task, students will consider a different context under which a system has one solution and compare this to the conditions under which a system has infinitely many solutions.

### Task 7: Pick Your Programs

Many cable television companies charge a flat rate for a cable TV package. As an alternative, some cable companies are starting to offer packages where customers pay a flat rate for access plus a fee per channel. Two of these companies are described below.

- **TV Party** charges a $40.00 flat rate plus $1.50 per channel.
- **Cable Club** charges a flat rate of $20.00 plus $3.00 per channel.

1. **Alicia** argues that since the cost per channel for **Cable Club** is twice the cost per channel of **TV Party**, but the flat rate is half of **TV Party**’s, then the companies will always cost the same amount for any number of channels. Do you agree or disagree with **Alicia**? Justify your reasoning using the equations and graphs.

2. Create a cable plan with a flat rate and a cost per channel that is always less expensive than **TV Party** and **Cable Club**. Explain how you know that your plan will always be less expensive.

3. Is it possible to create a cable plan that is less expensive than **TV Party** for any number of channels, but more expensive than **Cable Club** for any number of channels? Use words and equations to justify your answer.

### Common Core Content Standards

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<tr>
<th>Standards for Mathematical Practice</th>
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**Task 7: Pick Your Programs**

Many cable television companies charge a flat rate for a cable TV package. As an alternative, some cable companies are starting to offer packages where customers pay a flat rate for access plus a fee per channel. Two of these companies are described below.

- **TV Party** charges a $40.00 flat rate plus $1.50 per channel.
- **Cable Club** charges a flat rate of $20.00 plus $3.00 per channel.

1. **Alicia** argues that since the cost per channel for **Cable Club** is twice the cost per channel of **TV Party**, but the flat rate is half of **TV Party**’s, then the companies will always cost the same amount for any number of channels. Do you agree or disagree with **Alicia**? Justify your reasoning using the equations and graphs.

2. Create a cable plan with a flat rate and a cost per channel that is always less expensive than **TV Party** and **Cable Club**. Explain how you know that your plan will always be less expensive.

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### Essential Understandings

- The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.
- Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions.
- Because both the $x$-value and the $y$-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.
- Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions.

### Materials Needed

- Student reproducible task sheet
- Straight edge, grid paper/additional lined paper, graphing calculators (optional)
SET-UP PHASE
Please read the task individually, and then I’ll ask for a volunteer to read the task aloud. After reading:
Does anybody know how much cable costs per month? Does it vary depending on your plan? How many people who have cable watch all 100 channels (or do you only watch a few)? Does this mean you’re still paying for channels that you don’t even watch? How do the companies in the task price cable television? I’m going to allow several minutes of private think time to work independently before working with partners. Note that for the first question, you may use more than one representation (table, graph, or equation) to support your opinion about Alicia’s statement and note that for the second question, you are creating your own plan.

EXPLORE PHASE (SMALL GROUPTIME, APPROXIMATELY 10 MINUTES)

<table>
<thead>
<tr>
<th>Possible Student Pathways</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
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</thead>
<tbody>
<tr>
<td><strong>Group can’t get started.</strong></td>
<td>Can you tell me what Alicia is saying in your own words?</td>
<td>Do you think Alicia is correct? How can we test her conjecture?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(If necessary) Can you create a table that will test her conjecture?</td>
</tr>
<tr>
<td><strong>Writes equations.</strong></td>
<td>How did you determine these equations? How does each equation represent the problem situation?</td>
<td>How can you be sure that these equations do not always have the same solutions? Can you determine when they do have the same solution?</td>
</tr>
<tr>
<td>TV Party: ( y = 40 + 1.50x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cable Club: ( y = 20 + 3x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Since the cable company packages can be written as different equations, they are not the same and won’t charge the same for all channels.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Less expensive company: ( y = 19 + 3x )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>They will always cost $1 less than Cable Club.</td>
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</tbody>
</table>
## Possible Student Pathways

### Creates a table of values.

<table>
<thead>
<tr>
<th># of Channels</th>
<th>TV Party ($)</th>
<th>Cable Club ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41.50</td>
<td>23.00</td>
</tr>
<tr>
<td>2</td>
<td>43.00</td>
<td>26.00</td>
</tr>
<tr>
<td>3</td>
<td>44.50</td>
<td>29.00</td>
</tr>
<tr>
<td>4</td>
<td>46.00</td>
<td>32.00</td>
</tr>
</tbody>
</table>

They don’t charge the same amount for these numbers of channels. Alicia is wrong.

### Creates a graph.

Can you explain your graph? How does this represent the problem situation? Does this support or refute Alicia’s claim?

How can you create a plan/graph that is always less expensive? Will the lines ever intersect? Explain.

### Explains using slope and $y$-intercept (no graph).

In order for a company to be less expensive all the time, they have to start lower and have a lower cost per channel than the other two. So, they could charge a $10 access fee and $1.49 per channel.

Why do they have to charge less both for access and per channel if they want to be less expensive all the time?

How will this appear graphically and in an equation?

### Finishes early.

How did you determine the less expensive plan?

What rule describes all plans that will always be less expensive than the two we have? How can you justify your response?

### Assessing Questions

What does your table of values represent? How does this support/refute Alicia’s claim?

If the table is extended further, will Cable Club ever become more expensive than TV Party? Why or why not?

### Advancing Questions

© 2013 University of Pittsburgh – Eighth Grade Set of Related Lessons: Understanding and Solving Systems of Linear Equations
SHARE, DISCUSS, AND ANALYZE PHASE

EU: The solution to a system of linear equations in two variables can be represented graphically by the point(s) of intersection of the lines representing the solutions to each of the equations in the system because that (those) intersection point(s) make(s) all of the equations true statements or satisfies all of the equations in the system simultaneously.

EU: Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions.

- I noticed groups thinking about this problem in many different ways. Who can share their initial thinking about this problem? (We chose a couple values for the number of channels and got different outputs for the cost customers will pay. This means that Alicia is wrong. The cost is not always the same.)
- Who can restate, in your own words, what this group said?
- Who can show how they used a graph to make sense of this problem?
- Let’s take a closer look at how they used graphs to represent this problem situation. Who can tell us about how the graph helped you think about Alicia’s argument? (We created a table of values for each plan and then graphed each company’s plan on the same grid. You can see when you extend the line that the two lines intersect only once.)
- What does the point of intersection represent in this problem? Why is their statement “only once” important? (It represents the number of channels for which both companies charge the same amount of money. This happens for only one number of channels.)
- The point of intersection on the graph does represent the number of channels for which both companies charge the same amount of money. Yesterday it represented the number of weeks at which both girls saved the same amount of money. It is important to realize that, just as a point has both an x- and y-value, so does the solution to a system of equations have both an x- and y-value. As a result, when we discuss what a point of intersection means in a context, we must mention both the x- and y-values (Marking)
- If we extend the graph far enough, will there be another point of intersection? Why or why not?
- I’m hearing that they will never intersect more than once (Marking), but who can explain why? (After they intersect once, the lines continue with one always at a higher rate, so the other one can never catch up.)
- Alicia thought that the companies will always cost the same amount for any number of channels. How does the graph that this group created convince us that Alicia is incorrect? (It only costs the same amount of money once.)
- What, then, will the graph of plans that always cost the same amount for any number of channels look like? (They must have the same graphs and therefore, the same equations.)
- Who can explain why the graphs must be the same in his or her own words? (Every x-value will have the same y-value.)
- So does that mean x = y? Who can say more or add on to help clear this issue up? (No. x does not have to be equal to y. But, if the graphs show that cost is the same amount for any number of channels for both companies, then whatever the cost is for x channels at TV Party, the cost will be the same at Cable Club. So all the (x, y) pairs for both plans will be the same.)

continued on next page
• What will that look like in a table? Graph? Equation? *(The tables will be the same. So will the graphs. So will the equations.)*
• Can someone add on? *(Each input has to give the same output. This only happens if the tables and equations are the same. The same equations would produce the same graphs.)*
• Interesting. We noted when we examined the To Meet or Not to Meet? task that if two equations are equivalent, then their graphs coincide or produce one and the same line. Now we are saying the “backwards” or converse of that statement is also true. If two lines coincide, then their equations must be the same or equivalent. *(Marking)* So we discussed this idea graphically, and the class is getting into the equations as well, so let’s discuss equations.

EU: Because both the $x$-value and the $y$-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.

• Let’s go back a moment to the TV Party and Cable Club plans. Group C, you used equations to examine this problem. How does the point of intersection relate to your equations? *(The point of intersection IS the solution. If you plug the $x$-value of the point of intersection into both of our equations, you will get the same $y$-value.)*
• Is it possible for these equations to share another solution? Why or why not? *(No. In the last task we discussed how if the slopes and $y$-intercepts are different, then they will only have one point of intersection, or one solution.)*
• This explains why graphically, but algebraically how do we know that only one solution exists?
• Who can explain the algebraic solution? *(I solved the equation $40 + 1.50x = 20 + 3x$ by subtracting 20 from both sides, subtracting 1.50x from both sides, then dividing by 1.5.)*
• Who can explain this equation? Why does this equation represent the problem situation? *(TV Party: $y = 40 + 1.50x$ because 40 is the flat rate and 1.50 is the charge per channel. Cable Club is $y = 20 + 3x$ because it has a flat fee of 20 and a rate of 3 per channel. Since they are both set equal to $y$, we can compare them by setting them equal to each other.)*
• Who can explain using another representation why we set $40 + 1.50x = 20 + 3x$? *(Since we are looking for the point of intersection, we are looking for $y$-values that are the same. So the expressions that represent those $y$-values must be the same.)*
• What does this solution represent in the context of this problem? *(The number of channels for which both companies will charge the same amount.)*
• So to summarize a couple important points: when we examine these companies using equations, we notice that the companies are represented by equations with different rates of change and different $y$-intercepts. Therefore, when we solve them algebraically we can expect only one solution. We expect the graphs of the equations to intersect only once. Therefore, Alicia is incorrect in her thinking. *(Recapping)*
• As we create a new plan in question 2, I want you to think about what the graph/equation/tables of values must look like if the companies never charge the same amount for a given number of channels. Consider the exploration that we did in the To Meet or Not to Meet? task and explain how we can create a cable company so that it will never cost the same amount as TV Party or Cable Club. *(Challenging)*
EU: Parallel lines have no points in common. Therefore, a system of linear equations representing distinct parallel lines has no solutions.

- Let’s take a look at several possible companies that were created for question 2 and create a running list of similarities and differences in these plans that will always be less expensive than TV Party and Cable Club.
- It looks like this less expensive company, Company C, represented by \( y = 19 + 3x \), is pretty close in cost to Cable Club. Who can convince me that it will not eventually become more expensive than Cable Club? (It will always be less expensive because it’s increasing at the same rate but starting at a dollar less.)
- Who can add on to this and explain what the graphs will look like? (The lines will cross the y-axis at different places but go up at the same rate.)
- Does that mean that the system of equations has no solution? (The system formed by Cable Club and Company C has no solutions. But the system formed by Company C and TV Party has solutions.)
- Okay. Looking at just Cable Club and Company C, I’m hearing you say that if the company has a lower starting cost, but the same rate of change, then the graphs of their plans will keep going up at the same rate and never intersect. (Revoicing) This is an important point—what is the word for lines that never intersect?
- Who can tell me how many solutions parallel lines will have and what this means in the context of this problem?
- How do you know, just by looking at the equations, that the Company C plan and the TV Party plan must have solutions?
- Will the Company C plan, \( y = 19 + 3x \), always be less expensive than TV Party? How do you know? (Challenging)
- Let’s try another plan, Company D, \( y = 19 + 1.50x \). How can we tell, just by looking at the equations, that the system formed by this plan and the Cable Club plan must have a solution?
- If we were to graph the lines, would they ever intersect? Why or why not? (They will never cost the same amount, but if you look at the graphs, they intersect way over here.)
- Who agrees, disagrees, or can add on? (Well, they intersect over here, where x is negative. So I guess that means they will never cost the same.)
- Say more. (x represents the number of channels.)
- Will someone who understands and agrees say more in your own words? (We can see that the graphs intersect, so there is a solution to the system. But the solution is where x, the number of channels, is negative. You can’t have a negative number of channels. So the point does not have meaning in the context of the problem, even though it exists.)
- Anyone else? (They’re intersecting in a different quadrant. The slopes and y-intercepts are different, so like we agreed in the last task, there must be a solution. However, the solution isn’t positive.)
- I’m hearing that this line (pointing) and the line representing Cable Club will have a point of intersection because the slopes and y-intercept are different, but in a quadrant where the intersection point makes no sense in the context of the problem. Meanwhile, in the first quadrant, where the context does make sense, we can see that the plan for Company D is always less expensive. (Recapping)
- Who can confirm this relationship algebraically? Explain.
EU: Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions.

EU: Because both the $x$-value and the $y$-value are the same at the point of intersection, use of the properties of equality on the system yields algebraic methods (substitution and the addition/elimination) for finding the solution to the system, or recognition that the system has no or an infinite number of solutions.

- One last check. Let’s have a group show their algebraic solution for two lines that are parallel and another group show their algebraic solution for two lines that intersect once and see how they are the same/different.
- We know that these two lines are parallel. What happened when the group tried to solve this algebraically? Why is that? Will this always be the case? *(Because the slopes are the same and the $y$-intercepts are different, when you try to solve for $x$, the $x$'s subtract to zero and the two constants are left equal to each other, making an untrue statement.)*
- So I’m hearing that since the graphs don’t intersect we also cannot get an algebraic solution. *(Marking)* How is this the same/different than when they do intersect? *(You get one solution.)*
- Let’s add what we just noted to our chart, under algebraic strategies. If, when we set the two linear expressions equal to one another, the expressions have different slopes, we will expect one solution to the system. And graphically, we expect the lines to intersect in one point—so let’s connect those ideas with a line. Meanwhile, if the two expressions have the same slope but different intercepts, we expect no solutions. Instead, we get a false statement of equality. Graphically, we see parallel lines. Let’s connect those ideas with a line, also. *(Marking)* Hmm… I wonder what we should expect when the two equations coincide? *(Challenging)* Who will take up that challenge to explore tonight and be prepared to share their ideas tomorrow?
- So then use what we’ve learned to answer the last question: is it possible to create a cable plan that is less expensive than TV Party for any number of channels, but more expensive than Cable Club for any number of channels? Use words, graphs, or equations to justify your answer.
- What generalization can we make about when a system of two linear equations will have no solutions? One solution?

**Application**

TV Party is represented by the equation $y = 40 + 1.50x$. Write an equation of a line parallel to TV Party. Write an equation of a line that intersects TV Party once. Explain your reasoning.

**Summary**

See the last generalizing question in the Share, Discuss, and Analyze phase of the lesson.

**Quick Write**

Explain how to identify from the graph, table, or equations when a system of equations will have no solutions, one solution, or infinitely many solutions.
Support for students who are English learners (EL):

1. Students who are English learners may be unfamiliar with the context. During the set-up be sure to describe cable television and how it differs from “regular” television.

2. Continue to add to the English term/appropriate native language term for math terms relevant to the unit as is necessary (here, “quadrant”).

3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can rehear and reprocess the day’s key ideas. Ask them to point to parallel lines when they are discussed, and to the point of intersection as it is discussed. Have them trace with their finger the lines that coincide as they are discussed.

4. Continue the poster titled, Strategies for Solving Systems of Equations. Add advantages and disadvantages of the strategy as those ideas emerge in the discussion, and ask students who are English learners to step back and notice similarities, differences, and/or patterns.
Name_________________________________________________________

**Still Stretching: Yoga Plans Revisited**

*Recall that Flexible Yoga charges a base rate of $35 to join, plus $4.50 per class and Stretch-It-Out charges a base rate of $27.50 to join and $5.25 per class.*

1. The cost of a third company, Poses-R-Us, is represented below as a line parallel to the line representing the cost of Stretch-It-Out.

![Graph showing the cost of Poses-R-Us and Stretch-It-Out](image)

Identify the equation(s) or description(s) below that may represent the cost of Poses-R-Us.

a. \( y = 27.50 + 6x \)
b. \( y = 30 + 5.25x \)
c. An equation with the same base rate as Stretch-It-Out but a different charge per class
d. An equation with the same charge per class as Stretch-It-Out but a different base rate

Explain your reasoning.
2. The cost of taking classes at a fourth yoga studio is the same as Flexible Yoga for only one number of classes. Write an equation that may represent the cost of taking classes at this company. Explain your reasoning.

3. Do you agree or disagree with the following statement: “If two of the yoga studios charge a fixed amount per class and cost the same total amount for the first and second classes, then they will cost the same amount for any number of classes.” Explain your reasoning.
**Still Stretching: Yoga Plans Revisited**

**Rationale for Lesson:** Students solidify their understanding of equations that have 0, 1, or infinitely many solutions using multiple representations.

**Task 8: Still Stretching: Yoga Plans Revisited**

Recall that Flexible Yoga charges a base rate of $35 to join, plus $4.50 per class and Stretch-It-Out charges a base rate of $27.50 to join and $5.25 per class.

1. The cost of a third company, Poses-R-Us, is represented below as a line parallel to the line representing the cost of Stretch-It-Out.
   - Identify the equation(s) or description(s) below that may represent the cost of Poses-R-Us.
     - a. $y = 27.50 + 6x$
     - b. $y = 30 + 5.25x$
     - c. An equation with the same base rate as Stretch-It-Out but a different charge per class
     - d. An equation with the same charge per class as Stretch-It-Out but a different base rate
   - Explain your reasoning.

2. The cost of taking classes at a fourth yoga studio is the same as Flexible Yoga for only one number of classes. Write an equation that may represent the cost of taking classes at this company. Explain your reasoning.

See student paper for complete task.

**Common Core Content Standards**

- **8.EE.C.8a** Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

- **8.EE.C.8b** Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. *For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6.*

**Standards for Mathematical Practice**

- MP1 Make sense of problems and persevere in solving them.
- MP2 Reason abstractly and quantitatively.
- MP3 Construct viable arguments and critique the reasoning of others.
- MP4 Model with mathematics.
- MP5 Use appropriate tools strategically.
- MP6 Attend to precision.
- MP7 Look for and make use of structure.
- MP8 Look for and express regularity in repeated reasoning.
<table>
<thead>
<tr>
<th>Essential Understandings</th>
<th>Materials Needed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution.</td>
<td>Student reproducible task sheet</td>
</tr>
<tr>
<td>Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions.</td>
<td>Straight edge, grid paper/additional lined paper, calculators (optional)</td>
</tr>
<tr>
<td>Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions.</td>
<td></td>
</tr>
</tbody>
</table>
SET-UP PHASE

I’d like a student to read the task aloud while everybody else reads along silently. As you are reading, highlight key mathematical information that will be useful in the problem. What is yoga? How do the yoga studios in this problem charge their customers? What is the difference between a base rate and a cost per class? Work on the problems individually for about ten minutes before sharing your ideas with the group.

EXPLORE PHASE (SMALL GROUP TIME, APPROXIMATELY 10 MINUTES)

### Possible Student Pathways

<table>
<thead>
<tr>
<th>In question 1, reasons through the problem using the cost per class (slope) and base rate (y-intercept).</th>
<th>Assessing Questions</th>
<th>Advancing Questions</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the graph, how do you know the cost per class is the same but the base rates are different?</td>
<td>How are these differences represented in equations?</td>
<td></td>
</tr>
<tr>
<td>For example: the cost of a class at Poses-R-Us is the same as Stretch-It-Out, but the base rates must be different.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| In question 1, creates a table of values (may be two separate tables). |  |
| Tell me about how you created this table of values. What does this tell you about the costs of the yoga studios? | How can you represent each studio with equations? What will the terms in each equation represent? |

<table>
<thead>
<tr>
<th>Classes</th>
<th>Flexible Yoga (Cost in dollars)</th>
<th>Poses-R-Us (Cost in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>35.00</td>
<td>40.00</td>
</tr>
<tr>
<td>1</td>
<td>39.50</td>
<td>44.50</td>
</tr>
<tr>
<td>2</td>
<td>44.00</td>
<td>49.00</td>
</tr>
<tr>
<td>3</td>
<td>48.50</td>
<td>53.50</td>
</tr>
<tr>
<td>4</td>
<td>53.00</td>
<td>58.00</td>
</tr>
<tr>
<td>5</td>
<td>57.50</td>
<td>62.50</td>
</tr>
</tbody>
</table>
### Possible Student Pathways

#### Makes sense of question 3 using a table of values (may be two separate tables).

<table>
<thead>
<tr>
<th>Classes</th>
<th>Flexible Yoga (Cost in dollars)</th>
<th>Fourth yoga studio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>39.50</td>
<td>39.50</td>
</tr>
<tr>
<td>2</td>
<td>44.00</td>
<td>44.00</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

They will cost the same amount for other values because they are going up at the same rate.

#### Makes sense of question 3 using a graph.

For example:

![Graph](image)

Only one line can go through two points.

#### Finishes early.

### Assessing Questions

Can you tell me about your table of values? How do you know they will cost the same amount for 0, 3, 4, 5, etc. classes?

### Advancing Questions

It looks like Flexible Yoga and this other studio will cost the same amount for any class. Why will this be the case for any two studios?

Tell me about your graph. How does this graph help you make sense of this problem?

How can you be sure that the two lines will continue on top of each other in this way? How will this appear in the equation and table of values?

How do the equation and graph of two parallel lines compare? Intersecting lines?

Can you write a second equation that will intersect the line representing Flexible Yoga only once? Explain your reasoning.
SHARE, DISCUSS, AND ANALYZE PHASE

EU: Parallel lines have no points in common. Therefore, a system of two linear equations representing distinct parallel lines has no solutions.
EU: Linear equations representing the same line have infinitely many points in common. Therefore, a system of two linear equations representing the same line has infinitely many solutions.

- Who can explain how they made sense of the lines? What do you know about these lines? (The lines will never intersect.)
- Who agrees or disagrees? Why? (They are going to keep going and never touch.)
- What is the term for lines that never touch? Then I’d like someone to explain how you know these lines will never touch. (Parallel. They are parallel because they keep increasing at the same rate.)
- How do you know that they are increasing at the same rate? What did you do to test this theory? (We formed slope triangles for each line. Both seem to be the same. So their slopes are the same.)
- Seem to be the same? Explain more. (The rises and runs seem to be the same lengths. We can’t read exact points, but they are close.) Is it enough to say they are parallel because they increase at the same rate? In other words, is it possible for two lines to increase at the same rate and not be parallel? (No…not unless they are the same line.)
- This is important. The lines will never intersect because they are increasing at the same rate. So if they have the same rate of change, unless they are the same line, then they will be parallel. (Marking)
- So what does this all mean in the context of this problem? (The lines representing the two yoga studios will never intersect.)
- Who can add on? (They will never cost the same amount for the same number of classes.)
- So how does the idea that the lines are parallel affect the possible equation representing Poses-R-Us? (It must have the same rate and a different y-intercept as the other line, so b and d are correct.)
- Okay. So parallel lines have the same slope and different y-intercepts. (Marking)
- I heard somebody say an important point earlier about the difference between the equations of parallel lines and equations of lines that are the same. Who can restate this in their own words? (Parallel lines have the same slope and a different y-intercept while equations that are the same line obviously have the same slope and y-intercept.)
- Okay, but what do the slopes and y-intercepts determine about the number of solutions the lines have in common? (Parallel lines have no points in common while lines that are the same have all of their points in common.)
- How many points in common do coinciding lines have? (Challenging) (Infinitely many.)
- So then how many points need to be in common in order for all of the points to be in common, as long as the rate of change remains constant? (Just two.)
- Who agrees or disagrees? Why?
- Group A, show us how you were convinced that two points are enough on the graph.
- So then if the yoga studios cost the same amount for one class and then again, the same amount for two classes, will they cost the same amount for all of them?
- So when we have a system of equations represented graphically, two possibilities are that lines have no points in common, in which case they’re parallel. The equations of parallel lines have the same rate of change and different y-intercepts. Or they can have infinitely many points in common, in which case they are directly on top of each other (same line). They coincide. (Recapping)
EU: Two distinct lines will intersect at one point if and only if they do not have the same slope. Therefore, a system of two linear equations representing distinct lines with different slopes has one solution.

- Who can explain another possibility for two distinct lines? *(They may intersect in just one point.)*
- Who agrees or disagrees?
- Why consider these three graphic possibilities? What do they tell us about systems of equations? *(The number of solutions.)*
- So what are the three possibilities for the number of solutions a system of two linear equations may have? *(They may have one, zero, or infinitely many.)*
- So I’m hearing from our conversations that three possibilities exist: The lines intersect once, not at all, or infinitely many times. Those graphic possibilities tell us that a system of two linear equations in two variables has one, none, or infinitely many solutions. For the most part, mathematicians are primarily interested in systems of equations that have exactly one solution. *(Revoicing and Recapping)*
- I just graphed Flexible Yoga. Who can describe Flexible Yoga’s cost per class and their base rate, and explain how you know this from the graph? *(They charge $35 base rate and $5.25 per class.)*
- In question 2 you are asked to determine the equation for a fourth yoga studio that costs the same amount once. Who can explain what the graph of fourth studio might look like? Who can show a second possibility? A third? A fourth?
- Is it possible to sketch one with the same y-intercept? *(Challenging)*
- Who can generalize about the characteristics of these lines that are shown here on the graph? *(They’re all increasing at a different rate.)*
- Who can restate this, using the graph in your explanation?
- How does this appear in the equations? Who can write several examples of equations that represent a studio that costs the same amount as Flexible Yoga only once?
- What does the point of intersection represent? *(A point they have in common.)*
- Who can add on? *(This is a solution to both equations.)*
- In the context of the problem what does this mean?
- Do we even need the graph to know that these equations will intersect $y = 35 + 5.25x$ only once? *(No, they simply have to have a different rate of change.)*
- I’m hearing that lines with different rates of change will always intersect only once and this intersection represents a solution to both equations. *(Recapping)*

<table>
<thead>
<tr>
<th>Application</th>
<th>No application.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Summary</td>
<td>Describe the difference in the graphs and equations of pairs of lines that have one solution in common, no solutions in common, and infinitely many points in common.</td>
</tr>
<tr>
<td>Quick Write</td>
<td>No Quick Write for students.</td>
</tr>
</tbody>
</table>
Support for students who are English learners (EL):
1. Students who are English learners may be unfamiliar with the context. If necessary, during the set-up, revisit the pictures and/or video from Task 2, Yoga Plans.
2. Revisit the English term/appropriate native language term for math terms relevant to the unit. Make sure connections are made between these terms and the chart of strategies.
3. During whole class discussions, ask students to repeat key ideas and to put ideas in their own words so that all students can rehear and reprocess the day’s key ideas. Ask them to point to parallel lines when they are discussed, and to the point of intersection as it is discussed. Have them trace with their finger the lines that coincide as they are discussed.
4. Revisit the poster titled, Strategies for Solving Systems of Equations. Ask students to step back and notice similarities, differences, and/or patterns. Connect the ideas in this chart to the English term/appropriate native language terms.